RFID & Item-Level Information Visibility

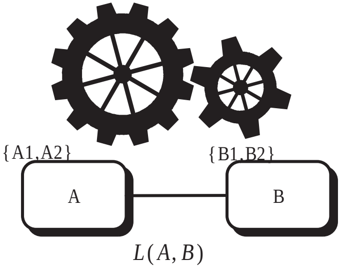
Zhou, Wei

Department of Information System & Operations Management
University of Florida
Gainesville, Florida 32601

wzhou@ufl.edu

December 13, 2008

1. Manufacturing an Engine



- $B_1 = 10\text{cm}$ & $B_2 = 10.2\text{cm}$
- $T_1 = 10.2\text{cm}$ & $T_2 = 10.4\text{cm}$
- $L(B_1, T_1) = L(B_2, T_2) = 10$ years
- $L(B_1, T_2) = L(B_2, T_1) = 9$ years

Link: Shandong Province Adopts RFID For Quality Control

Building one engine:

- Without RFID visibility
  - $E[L] = E[E(B|T)] = 9.5$ years.
- With RFID visibility
  - $L = \max\{L(B_1, T_1), L(B_2, T_2), L(B_1, T_2), L(B_2, T_1)\} = 10$ years

Building two engine:

- Without RFID visibility
  - 19 years.
- With RFID visibility
  - 20 years

2. Replenish Retail Shelf

- There's only one client in the store
- The client has 50 % possibility to buy a bottle of shampoo
- Original stock: 20 bottles
- Remaining number of shampoo can be any number in $\{0, 1, 2, \cdots, 20\}$
- If the shelf is empty, the client leaves without buying the shampoo.

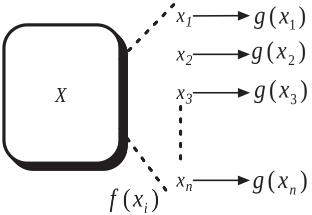
- Without RFID visibility
  - Expected sale $= 50\% \cdot \frac{20}{21} = 0.476$
- With RFID visibility
  - store clerk replenish the shelf once it's empty
  - Expected sale $= 0.5$ bottle

Notations

- $\{X_1, X_2, \cdots, X_m\}$: m components
- $\{X_i | x_{i1}, x_{i2}, \cdots, x_{in_i}\}$: tagged information of cases in a component
- $X \sim f_x(X)$: statistical property of the tagged information
- $y_i = g(x_i)$: production function of x_i
- O: the outcome without RFID visibility
- $\tilde{O}$: the outcome with RFID visibility

Model Setup

- $X \sim f_x(X)$
- $\{X | x_1, x_2, x_3, ..., x_n\}$
- $y = g(x_i)$
- $O : O = E[g(X)]$
- $\tilde{O} : \tilde{O} = \max\{g(X)\}$



The p.d.f. of the sample without information visibility is:

$$f_y(y) = f_x(g^{-1}(y))g\prime(y)$$

The p.d.f. of the sample with information visibility is:

$$f_n^y(y) = n \cdot f_x(g^{-1}(y))g\prime(y) \cdot \left(\int_{-\infty}^{y} f_x(g^{-1}(y))g\prime(y)dy \right)^{n-1}$$

The difference (benefit) of having information visibility is:

$$\begin{aligned}\delta &= \tilde{O} - O \\ &= \int_y y u\prime (n u^{n-1} - 1) dy\end{aligned}$$

where

$$u = \int_{-\infty}^{y} f_x(g^{-1}(y) g\prime(y) dy$$

Best k outcomes

The p.d.f. of the kth best production is:

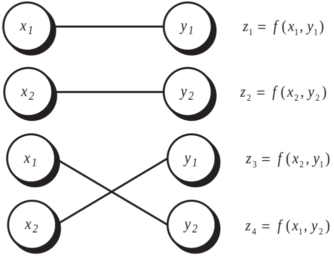
$$f^{y}_{k:n}(y) = \frac{n!}{(k-1)!(n-k)!} u^{k-1}(1-u)^{n-k} f_y(y)$$

Hence the benefit of having information visibility is:

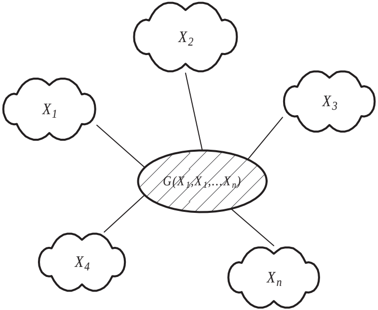
$$\begin{aligned}\delta &= \sum_{i=1}^{k} (E[y_{i:n}] - E[y]) \\ &= \int_y (nF_{binomial(n-1,u)}(k) - k) \cdot y f_y(y) dy\end{aligned}$$

Setup

Two components:



n components:



Assume that we have m components: $\{X|X_1, X_2 \cdots X_m\}$

$$\{X_1|x_{11}, x_{12}, x_{13} \cdots x_{1n_1}\}$$
$$\{X_2|x_{21}, x_{22}, x_{23} \cdots x_{2n_2}\}$$
$$\vdots$$
$$\{X_m|x_{m1}, x_{m2}, x_{m3} \cdots x_{mn_m}\}$$

$X_1, X_2 \cdots X_m$ follow joint distribution

$$f_{X_1, X_2 \cdots X_m}(X_1, X_2 \cdots X_m)$$

- $Y = g(X_1, X_2 \cdots X_m)$
- The c.d.f. of the sample with information visibility is:

$$F_y^N(y) = Pr[g(X_1, X_2 \cdots X_m) \leq y] = \int \cdots \int_{g(X_1, X_2 \cdots X_m) \leq y} f_{X_1, X_2 \cdots X_m}(X_1, X_2 \cdots X_m) dX_1 dX_2 \cdots dX_m$$

- The difference (benefit) with information visibility is:

$$\delta = \int_y n_1 n_2 \cdots n_m \cdot u\prime(u^{n_1 n_2 \cdots n_m - 1} - 1) y dy$$

where

$$u = \int \cdots \int_{g(X_1, X_2 \cdots X_m) \leq y} f_{X_1, X_2 \cdots X_m}(X_1, X_2 \cdots X_m) dX_1 dX_2 \cdots dX_m$$

The k best outcomes

$$f^{y}_{k:N}(y) = \frac{N!}{(k-1)!(N-k)!} u^{k-1}(1-u)^{N-k} f_y(y)$$

$$\begin{aligned}\sum_{i=1}^{k} E[y_{i:N}] &= \sum_{i=1}^{k} \int_y y \cdot \frac{N!}{(i-1)!(N-i)!} u^{i-1}(1-u)^{N-i} f_y(y) dy \\ &= \int_y nF_{binomial(N-1,u)}(k) \cdot y f_y(y) dy\end{aligned}$$

Hence the benefit of having information visibility, δ is:

$$\delta = \int_y (NF_{binomial(N-1,u)}(k) - k) \cdot y f_y(y) dy$$

Theorem (1)

The lower bound for δ is:

$$\delta \geq 0$$

.

Theorem (2)

When $k \leq n$, δ is upper bounded as:

$$\delta \leq \int_y (ne^{-2\frac{(n-k)^2}{n}} - k) \cdot yf_y(y)dy \leq \int_y (n-k) \cdot yf_y(y)dy$$

Proof.

Hoeffding's inequality & Chernoff's inequality:

$$F_{binomial}(k; n, u) \leq e^{-2\frac{(n-k)^2}{n}} \leq 1$$

□

Theorem (3)

If $k = n$, $\delta = 0$

Theorem (4)

Larger the sample space variance, larger the benefit δ.

Theorem (5)

if Y *is positive,* δ *is a monotonic increasing function of* n.

Theorem (6)

If all the resources are used, the benefit of having information visibility is zero if there's only one component; the benefit is greater than zero if there are multiple components.

Control Function

$$\psi(X) = \begin{cases} 19 & \text{if } X = 0 \\ X & \text{otherwise} \end{cases}$$

$$X \sim \psi(X)$$
$$\Downarrow$$
$$Y = g(\psi(X))$$
$$\Downarrow$$
$$\delta = \Omega\left(k, X, \psi(X), g(X)\right)$$

Finite Horizon

Using Bellman's Method, the recursive function is:

$$V(X_{T-k}, k) = \max_{v_{T-k}} \{\delta_{T-k}(X_{T-k}, v_{T-k}) + V(X_{T-k+1}, k-1)\}$$
$$\equiv \delta_{T-k}(X_{T-k}, \psi_{T-k}(X_{T-k})) + V(\xi_{T-k}(X_{T-k}, \psi_{T-k}(X_{T-k}), k-1)$$

subject to:

1. $X_{T-k+1} = \xi_{T-k}(X_{T-k}, v_{T-k}, G(v_{T-k}))$
2. $X_0 = \tilde{X}_0$
3. $v_{T-k} = \psi_{T-k}(X_{T-k})$
4. $v_t \in \Theta$ for all $t = 0, 1, \cdots, T-1$

Infinite Horizon

$$V_t(X_t) = \max_{v_t} \left\{ \beta\delta(X_t, v_t) + V_{t+1}(X_{t+1}) \right\} \tag{1}$$

subject to:

1. $X_{t+1} = \xi_0(X_t, v_t)$
2. $X_0 = \tilde{X}_0$
3. $v_t = \psi(X_t)$

where β is the discount factor and $0 \leq \beta \leq 1$.

Sargent (1987) and Stokey (1989): starting from any bounded continuous W_0 will cause W to converge as $t \to \infty$.

Simulation of Fabrication Quality Control

$$
\begin{aligned}
X_1 &\sim \text{Uniform } (-0.5, 0.5) \\
X_2 &\sim \text{Uniform } (-0.5, 0.5) \\
G(x_1, x_2) &= e^{x_1 - x_2} \\
\psi(X) &= X
\end{aligned}
$$

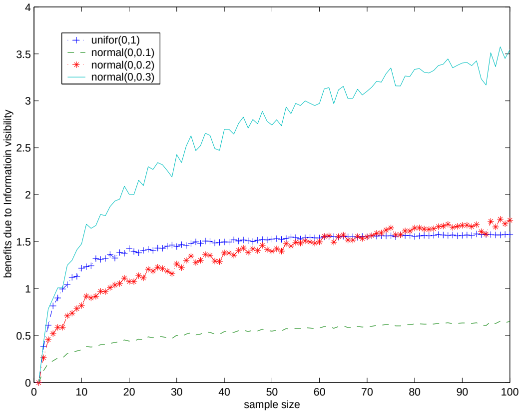


Figure: Benefit of having information visibility for fabrication quality control.

Analysis

The results show:

- The benefit of information visibility increases if the scale of the manufacturing increases
- The benefit function is concave and bounded.
- If the samples are more volatile, more is the benefit of having RFID information visibility.

Merci!