Role of access charges in the migration from copper to FTTH

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Abstract

We consider a horizontally and vertically differentiated duopoly model in order to analyze both intra- and inter-platform competition in an always corvered broadband access market (Copper-Copper, Copper-FTTH and FTTH-FTTH competitions). The model is purely static and does not address dynamic efficiency issues. It shows that the access charges play a significant role in the migration from copper to FTTH and in FTTH investment incentives, provided that consumers are segmented. In FTTH-infrastructure-based competition, investment incentives tend to increase with the copper access charge, while in FTTH-service-based competition, FTTH investment incentives are much more sensitive to the FTTH access charge than to the copper access charge. A comparison of FTTH-infrastructure-based and FTTH-service-based competition in terms of nationwide FTTH coverage and social welfare indicates that FTTH-infrastructure-based competition leads to a higher level of nationwide FTTH coverage and social welfare.

1 Introduction

The impact of broadband access regulations on competition and investment is a crucial issue, whose consequences are increasing with the rise of Next Generation Access Networks (NGN).

The tension between promoting competition and investment incentives has already been noted in economics literature. Laffont and Tirole, (2000) underline the trade-off between promoting competition, which increases social welfare once infrastructure is in place, and investment incentives, which are used to improve or simply maintain the infrastructure.

Kalmus and Wiethaus, (2007) have developed a model which describes the impact of access charges in different regulatory regimes.

Like Laffont and Tirole, they distinguish between infrastructures which are already in place and investments in new infrastructures, or upgrading existing infrastructures. In the case of existing infrastructures, they concluded that an access charge set at cost is the best way to maximize social welfare. For new infrastructures or upgrades, on the other hand, they concluded that an access charge set at cost is inefficient. The optimal access charge is higher than marginal costs.

In the first case, investments are not needed. The regulator simply maximizes the static efficiency of the infrastructure. In the second case, however, investments are needed and operators, including both existing operators and their rivals, must be encouraged to invest. A low access charge does not provide enough returns to investors and allows

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competitors to obtain access at low costs. They have also shown that investments and consumer surplus both increased with access charge.

More recently, Nitsche and Wiethaud (2010) have focused on the specific case of investments in NGNs. They concluded that the best regime was the "Fully Distributed Costs" (FDC) system, which maximizes the consumer surplus and "Risk sharing" in order to maximize investment. These regulatory regimes are more efficient than a simple access charge set at cost, known as the "Long Run Incremental Cost" (LRIC) system.

In recent decades, regulatory regimes, particularly in Europe, have been more focused on promoting competition than on investment incentives, in line with the "ladder of investment" theory of (Cave, 2006). Kalmus and Wiethaus (2007) explain the regulators' behavior:

"At the time when those investments were made, telecoms companies were usually state-owned monopolies. Therefore, regulators did not need to worry about negatively affecting investment incentive. From a consumer welfare point of view, they maximized consumer welfare by forcing access at cost"

This behavior has clearly made it possible to shake off competition from state owned monopolies, but the ladder of investment theory's efficiency has been called into question. A paper (Bourreau, Dogan, & Manant, 2009) noted that competitors seem to be stuck on the first rungs of the ladder of investment.

A growing current of thought, particularly in the USA, emphasizes the investment side and the overall dynamic effects of regulatory regimes. Bauer and Bohlin (2008) have observed a trend which moves from static to dynamic regulation. This new regulatory behavior is explained by the need to boost investments in order to upgrade the existing copper infrastructure.

Empirical studies highlight the impact of access charge regulation on investments. Waverman, Meschi, Reillier, and Dasgupta (2007) have shown that a low copper access charge encouraged intra-platform competition (DSL competition only) but hampered inter-platform competition (competition among DSL and cable or FTTx operators) and hence investment. They argue that, in the long run, its negative effects on inter-platform investment override its beneficial effects on intra-platform competition.

As Kalmus et al. have observed, investment, which allows inter-platform competition, acts over the long term. This is a dynamic effect and its impact depends on the investments effects on consumers' willingness to pay. Jeanjean (2010) highlights that the dynamic effect of investments depends on the potential of technological progress and is inversely proportional to the static efficiency of competition in maximizing welfare.

A paper by Brito, Pereira, and Vareda (2010) analyzed the incentives to invest according to the degree of improvement perceived by consumers between an old technology (copper) and a new technology (fiber). If only the incumbent operator can invest, when the improvement is non-drastic it may be induced to give access to the entering operator. Furthermore, when the improvement is small and non-drastic, a duopoly on the retail market is socially optimal, while when the improvement is non-drastic but large, a monopoly on the retail market is socially optimal. In this case the decrease in welfare caused by a decline in the level of competition is smaller than the decrease caused by the high level of the fiber access charge paid by the newcomer. When the improvement is drastic, the incumbent operator does not give access to the newcomer. The solution might be regulating the fiber access charge, but this may deter investments. If both firms can invest, but only one does, it is more likely that the entrant is the one which invests.

This paper aims to examine the trade-off between static efficiency, which is the advantage of cost-oriented access charges, and dynamic efficiency, which is the

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advantage of higher access charge. The paper's originality consists in dealing with intra-platform and inter-platform competition at the same time. It models competition between the incumbent operator, which owns the copper infrastructure, and its rival, which buys access from the incumbent. Both the incumbent and its rival may build a new FTTH infrastructure which allows enhanced services, which in turn increase consumers' willingness to pay. This new infrastructure may or may not be regulated by a fiber access charge. The model investigates both the level of the copper access charge and, if necessary, the fiber access charge.

Our investigation is divided into six sections. In section 2, a horizontally and vertically differentiated duopoly model is introduced (Shaked & Sutton, 1987). In section 3, we attempt to determine the FTTH investment incentives in FTTH-infrastructure-based competition. The interdependence of investment incentives and access charges in FTTH-service-based competition is demonstrated in section 4. Nationwide FTTH coverage, consumer surplus and social welfare are determined in section 5 and FTTH-infrastructure-based and FTTH-service-based competition are compared. Section contains our concluding remarks.

2 The linear model

In the following section, a two-player, four-offer model is introduced (Shaked & Sutton, 1987)

The two players are a vertically integrated firm -the incumbent which owns the copper network - and its rival. In order to analyze the role of the copper access charge, each operator is able to offer Internet access through either technology, but only the copper access charge is regulated. The incumbent manages the copper infrastructure, provides a copper offer, and possibly a FTTH offer, if it decides to invest. The rival provides a copper offer by paying an access charge to the incumbent and possibly, if it decides to invest, a FTTH offer by investing in a fiber infrastructure.

We assume that the consumer's utility to be connected to the network, whatever the technology, is V. The fiber network is supposed to provide higher quality than copper. We assume that the difference of utility between copper and fiber is θ. The incumbent incurs a marginal cost c for the copper offer for both the retail and wholesale markets. Both firms incur the same marginal cost c_f for the fiber offer.

The two firms are differentiated *à la Hotelling*, with the transportation cost *t*. The two technologies are differentiated vertically with the parameter *h*. We illustrate this in the following figure with two axes of differentiation on a two-dimensional surface.
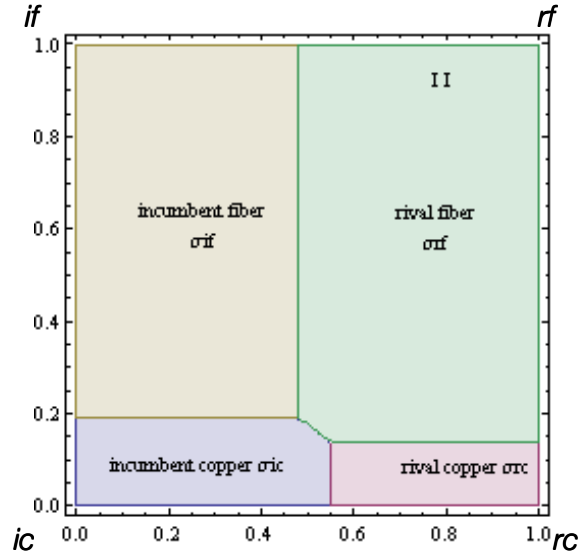
Figure 1 Market share of incumbent copper, rival copper, incumbent FTTH and rival FTTH

Competition between firms is represented by the horizontal axis and competition between technologies by the vertical axis. The incumbent is located at abscissa *0* and the rival at abscissa *1*. The copper technology is located at ordinate *0* and the fiber technology at ordinate *1*. The market size is normalized to *1, i.e.* $\sigma_{ic} + \sigma_{rc} + \sigma_{if} + \sigma_{rf} = 1$ where σ_{ic}, σ_{rc}, σ_{if}, σ_{ry} respectively represent the market share of incumbent copper, rival copper, incumbent fiber and rival fiber offers.

We assume that the constant *V* is high enough to ensure that the market is fully covered. θ corresponds to the average consumer valuation of the fiber technology. Consumers who purchase a fiber offer increase their utility by θ. Consumers who are located at ordinate y incur a disutility of $h(1-2y)$ so they increase their utility by $\theta - h(1-2y)$.

The utility of a consumer located at *(x,y),* who purchases an access is U_{ic} for the incumbent copper offer, U_{rc} for the rival copper offer, U_{if} for the incumbent fiber offer and U_{rf} for the rival fiber offer. The offer prices are denoted respectively: $p_{ic}; p_{if}; p_{rc}; p_{rf}$.

The different utilities are written:

$$
\begin{aligned}
U_{ic} &= V - p_{ic} - tx \\
U_{if} &= V - p_{if} + \theta - h(1-2y) - tx \\
U_{rc} &= V - p_{rc} - t(1-x) \\
U_{rf} &= V - p_{rf} + \theta - h(1-2y) - t(1-x)
\end{aligned}
$$

(2-1)

When *t* is high, operators are highly differentiated, when *t* is low, operators are highly substitutable and competition is fierce.
When *h* is high, the technologies are very vertically differentiated, and consumers see fiber and copper as very different. In other words, consumers are segmented (or distributed) according their access technology preference. When *h* is low, consumers all have a tendency to adopt the same behavior.

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Two cases are particularly relevant to understanding the impact of access charges on investment incentives.

The first case is $h = 0$. In this case, consumers all adopt the same behavior, and will all choose a fiber offer as soon as it is available because of the premium θ it provides.

The second case is $t = 0$, a perfect competition between firms.

In this study, we distinguish between two types of competition for FTTH offers:
- FTTH-infrastructure-based competition, only the FTTH network owner can propose FTTH offers
- FTTH-service-based competition, the FTTH network owner can or must offer wholesale FTTH access to competitors through a FTTH access charge

FTTH-infrastructure-based competition

Given that the copper access charge provides revenues for the incumbent and generates costs for the rival, profits expressions are as follow (in case of both firms invest in FTTH)

$$\pi_{ic} = (p_{ic}\text{-}c)\,\sigma_{ic} + (a_c\text{-}c)\,\sigma_{rc}$$
$$\pi_{rc} = (p_{rc}\text{-}a_c)\,\sigma_{rc}$$
$$\pi_{if} = (p_{if}\text{-}c_f)\,\sigma_{if}\text{-}f \qquad\qquad (2\text{-}2)$$
$$\pi_{rf} = (p_{rf}\text{-}c_f)\,\sigma_{rf}\text{-}f$$

Where c: marginal cost of copper access
a_c: copper access charge
c_f: marginal cost of FTTH
f: fixed cost of FTTH deployment

The prices at equilibrium are calculated as follows:

- The incumbent maximizes the sum of the profits $(\pi_i = \pi_{ic} + \pi_{if})$ with respect to p_{ic} and p_{if}
- The rival maximizes the sum of the profits $(\pi_r = \pi_{rc} + \pi_{rf})$ with respect to p_{rc} and p_{rf}

FTTH-service-based competition

The profit expressions differ depending on which player invests in FTTH. If the incumbent invests in FTTH and the rival proposes an FTTH offer by paying an FTTH access charge to the incumbent, the profit expressions become:

$$\pi_{ic} = (p_{ic}\text{-}c)\,\sigma_{ic} + (a_c\text{-}c)\,\sigma_{rc}$$
$$\pi_{rc} = (p_{rc}\text{-}a_c)\,\sigma_{rc}$$
$$\pi_{if} = (p_{if}\text{-}c_f)\,\sigma_{if} + (a_f\text{-}c_f)\,\sigma_{rf}\text{-}f \qquad\qquad (2\text{-}3)$$
$$\pi_{rf} = (p_{rf}\text{-}a_f)\,\sigma_{rf}$$

Where a_f: FTTH access charge

If the rival invests in FTTH and the incumbent proposes an FTTH offer by paying an FTTH access charge to the rival, the profit expressions become:

$$\pi_{ic} = (p_{ic}\text{-}c)\,\sigma_{ic} + (a_c\text{-}c)\,\sigma_{rc}$$
$$\pi_{rc} = (p_{rc}\text{-}a_c)\sigma_{rc}$$
$$\pi_{if} = (p_{if}\text{-}a_f)\,\sigma_{if} \tag{2-4}$$
$$\pi_{rf} = (p_{rf}\text{-}a_f)\,\sigma_{rf} + (a_f\text{-}c_f)\,\sigma_{if}\text{-}f$$

Determination of investment strategy

In order to determine the dominant FTTH investment strategy for the incumbent and the rival, a payoff table is created for the four situations, referred to as *NN*, *NI*, *IN* and *II*.

	No rival investment	Rival investment
	NN	*NI*
No incumbent investment	$\pi_i^{NN} > \pi_i^{IN}\ \&\ \pi_r^{NN} > \pi_r^{NI}$	$\pi_i^{NI} > \pi_i^{II}\ \&\ \pi_r^{NI} > \pi_r^{NN}$
	IN	*II*
Incumbent investment	$\pi_i^{IN} > \pi_i^{NN}\ \&\ \pi_r^{IN} > \pi_r^{II}$	$\pi_i^{II} > \pi_i^{NI}\ \&\ \pi_r^{II} > \pi_r^{IN}$

Table 1 Payoff table for *NN, NI, IN, II*

NN corresponds to a situation where nobody invests: only two copper offers exist on the broadband market. One copper offer is provided by the incumbent who owns the copper network and the other by the rival who buys the copper line by paying an access charge to the incumbent. *NN* is a dominant strategy when $\pi_i^{NN} > \pi_i^{IN}\ \&\ \pi_r^{NN} > \pi_r^{NI}$. This expression means that neither the incumbent nor the rival has incentives to invest alone.

NI corresponds to a situation where the rival invests alone: two copper offers and one FTTH offer (or two FTTH offers) exist on the broadband market in FTTH-infrastructure-based competition (FTTH-service-based competition). In FTTH-service-based competition, FTTH access can be offered by both firms: first by the rival who has invested in FTTH and then by the incumbent who buys FTTH access by paying an access charge to FTTH network owner. *NI* is a dominant strategy when $\pi_i^{NI} > \pi_i^{II}\ \&\ \pi_r^{NI} > \pi_r^{NN}$. This expression means that the rival has an incentive to invest alone while the incumbent has no incentive to invest when the rival invests.

IN corresponds to a situation where the incumbent invests alone: two copper offers and one FTTH offer (or two FTTH offers) exist on the broadband market in FTTH-infrastructure-based competition (FTTH-service-based competition). As for *NI*, in FTTH-service-based competition, FTTH access can be offered by both firms: first by the incumbent who has invested in FTTH and then by the rival who buys FTTH access by paying an access charge to the incumbent. *IN* is a dominant strategy

when $\pi_i^{IN} > \pi_i^{NN}$ & $\pi_r^{IN} > \pi_r^{II}$. This expression means that the incumbent has an incentive to invest alone while the rival has no incentive to invest when the incumbent invests.

II corresponds to a situation where both firms invest: two ADSL offers and two FTTH offers exist on the broadband market. The incumbent and the rival are therefore in competition for ADSL in terms of services and in competition for FTTH in terms of facilities. The incumbent and the rival both maximize the sum of their ADSL and FTTH profits. *II* is a dominant strategy when $\pi_i^{II} > \pi_i^{NI}$ & $\pi_r^{II} > \pi_r^{IN}$. This expression means that the incumbent and the rival are both encouraged to invest when the other is investing.

In the same manner, we are able to determine the FTTH investment strategy based on the actions and reactions of both firms in the four situations described above. We will call this maximum amount of investment $f_i^{\ j}$. With i, the firm $i \in \{I, R\}$ and j the number of firms which invest $j = 1$ if the firm invests alone and $j = 2$ when both firms invest.

	No rival investment	Rival investment
No incumbent investment	$f_i^1 = \pi_i^{IN} - \pi_i^{NN} < 0$ & $f_r^1 = \pi_r^{NI} - \pi_r^{NN} < 0$	$f_r^1 = \pi_r^{NI} - \pi_r^{NN} > 0$ & $f_i^2 = \pi_i^{II} - \pi_i^{NI} < 0$
Incumbent investment	$f_i^1 = \pi_i^{IN} - \pi_i^{NN} > 0$ & $f_r^2 = \pi_r^{II} - \pi_r^{IN} < 0$	$f_r^2 = \pi_r^{II} - \pi_r^{IN} > 0$ & $f_i^2 = \pi_i^{II} - \pi_i^{NI} > 0$

Table 2 Actions and reactions of the incumbent and the rival for *NN, NI, IN, II*

A positive f_i^1 means that the incumbent invests in FTTH when the rival does not (incumbent's investment action). A positive f_r^1 means that the rival invests in FTTH when the incumbent does not (the rival's investment action). A positive f_i^2 means that the incumbent invests in FTTH when the rival does (the incumbent's investment reaction). A positive f_r^2 means that the rival invests in FTTH when the incumbent does (the rival's investment reaction).

NN is a dominant strategy when $f_i^1 = \pi_i^{IN} - \pi_i^{NN} < 0$ & $f_r^1 = \pi_r^{NI} - \pi_r^{NN} < 0$. This expression means that no firm invests
NI is a dominant strategy when $f_r^1 = \pi_r^{NI} - \pi_r^{NN} > 0$ & $f_i^2 = \pi_i^{II} - \pi_i^{NI} < 0$. This expression means that the rival invests and the incumbent does not react, meaning that the rival invests alone.
IN is a dominant strategy when $f_i^1 = \pi_i^{IN} - \pi_i^{NN} > 0$ & $f_r^2 = \pi_r^{II} - \pi_r^{IN} < 0$. This expression means that the incumbent's investment action is confirmed and the rival's reaction is absent, meaning that the incumbent invests alone.

II is a dominant strategy when $f_r^2 = \pi_r^{II} - \pi_r^{IN} > 0$ & $f_i^2 = \pi_i^{II} - \pi_i^{NI} > 0$. This expression means that when a firm invests, whether it is the incumbent or the rival, the other reacts, meaning that both firms invest.

Which of the four situations **NN, NI, IN and II** is a dominant strategy? The answer depends on two parameters. In FTTH-infrastructure-based competition, it mainly depends on the copper access charge, a_c, and fixed FTTH deployment cost f. In FTTH-service-based competition, it mainly depends on the FTTH access charge, a_f and fixed FTTH deployment cost f.

This is a three-stage game. In the first stage, the regulator sets the copper access charge a_c. In the second stage, both players decide whether to invest in the fiber infrastructure. In the third stage, players compete on the retail price of copper and possibly fiber. The game is studied for a given area, with a given fixed FTTH infrastructure cost *f*, which is assumed to be equal for both firms. As usual, the game is solved by backward induction. By comparing each player's profits, with and without fiber investment, we are able to determine fiber investment incentives.

3 Model resolution in FTTH-infrastructure-based competition

Three cases are studied in this section. Subsection 3.1 analyses the case where copper and FTTH are seen as fully substitutable technologies with h=0. Subsection 3.2 analyses a perfect competition with t=0. Subsection 3.3 analyses imperfect competition in a segmented market with t > 0 and h > 0. The latter case is a generic case where we use numerical simulation to solve the model.

3.1 Fully substitutable technologies h=0

In this case, the utility functions in equation (2-1) can be rewritten as follow:

$$
\begin{aligned}
U_{ic} &= V - p_{ic} - tx \\
U_{if} &= V + \theta - p_{if} - tx \\
U_{rc} &= V - p_{rc} - t(1-x) \\
U_{rf} &= V + \theta - p_{rf} - t(1-x)
\end{aligned}
\tag{3-1}
$$

The equilibrium may be different depending on whether the incumbent or the rival invests or both invest.

3.1.1 No firm invests

In this case, fiber offers are not available, consumers can choose only between the incumbent and rival's copper offers.
 Market share is written:

$$\sigma_{ic} = \frac{1}{2} + \frac{p_{rc} - p_{ic}}{2t}$$

$$\sigma_{rc} = \frac{1}{2} + \frac{p_{ic} - p_{rc}}{2t}$$

(3-2)

Profits depend on access charge a_c and copper marginal cost c:

$$\pi_i^{NN} = \sigma_{ic}(p_{ic} - c) + \sigma_{rc}(a_c - c)$$

$$\pi_r^{NN} = \sigma_{rc}(p_{rc} - a_c)$$

(3-3)

A first order condition allows us to determine equilibrium prices:

$$p_{ic}^{NN} = t + a_c$$

$$p_{rc}^{NN} = t + a_c$$

(3-4)

and profits at equilibrium:

$$\pi_i^{NN} = \frac{t}{2} + a_c - c$$

$$\pi_r^{NN} = \frac{t}{2}$$

(3-5)

We can observe that the rival's profit does not depend on copper access charge because the rival can pass along the price burden and preserve its margin without reducing its market share. This is not possible when at least one fiber offer is available.

An increase in access charge increases the incumbent's profit and has no effect on the rival's profit.

$\frac{\partial \pi_i^{NN}}{\partial a_c} = 1$; $\frac{\partial \pi_r^{NN}}{\partial a_c} = 0$. This reduces the incumbent's incentive to invest when access charge increases.

3.1.2 Only the incumbent invests

In this case, the incumbent's consumers all choose the fiber offer while the rival's consumers all choose the copper offer. While all of its consumers have migrated towards fiber, the incumbent continues to receive revenues from the wholesale market.

Market share is written:

$$\sigma_{if} = \frac{1}{2} + \frac{p_{rc} - p_{if} + \theta}{2t}$$

$$\sigma_{rc} = \frac{1}{2} + \frac{p_{if} - p_{rc} - \theta}{2t}$$

(3-6)

$$\pi_i^{IN} = \sigma_{rc}(a_c - c) + \sigma_{if}(p_{if} - c_f)$$
$$\pi_r^{IN} = \sigma_{rc}(p_{rc} - a_c)$$

<div align="right">(3-7)</div>

Let us denote $\omega = c - c_f + \theta$ which represents the benefits of fiber as compared to copper. This is the marginal cost difference plus the consumers' utility difference. In order to ensure that the market is fully covered we assume $\omega \leq 3h$.

A first order condition leads to equilibrium prices:

$$p_{if}^{IN} = t + a_c + \theta - \frac{2\omega}{3}$$
$$p_{rc}^{IN} = t + a_c - \frac{\omega}{3}$$

<div align="right">(3-8)</div>

And profits at equilibrium:

$$\pi_i^{IN} = a_c - c + \frac{(3t + \omega)^2}{18t}$$
$$\pi_r^{IN} = \frac{(3t - \omega)^2}{18t}$$

<div align="right">(3-9)</div>

An increase in access charge increases the incumbent's profit and has no effect on the rival's profit.

3.1.3 Only the rival invests

In this case, the rival's consumers all choose the fiber offer while the incumbent's consumers all choose the copper offer. The incumbent thus no longer receives revenues from the wholesale market.

Market shares are written:

$$\sigma_{ic} = \frac{1}{2} + \frac{p_{rf} - p_{ic} - \theta}{2t}$$
$$\sigma_{rf} = \frac{1}{2} + \frac{p_{ic} - p_{rf} + \theta}{2t}$$

<div align="right">(3-10)</div>

$$\pi_i^{NI} = \sigma_{ic}(p_{ic} - c)$$
$$\pi_r^{NI} = \sigma_{rf}(p_{rf} - c_f)$$

<div align="right">(3-11)</div>

A first order condition leads to equilibrium prices:

$$p_{ic}^{NI} = t + c - \frac{\omega}{3}$$

$$p_{rf}^{NI} = t + c_f + \frac{\omega}{3}$$

<div style="text-align: right">(3-12)</div>

And profits at equilibrium:

$$\pi_i^{NI} = \frac{(3t - \omega)^2}{18t}$$

$$\pi_r^{NI} = \frac{(3t + \omega)^2}{18t}$$

<div style="text-align: right">(3-13)</div>

An increase in access charge has no effect either on either firm.

3.1.4 Both firms invest

In this case, consumers all choose the fiber offer, no matter which firm they choose, and as in the previous case, the incumbent receives no revenues from the wholesale market.

Market shares and profits are written:

$$\sigma_{if} = \frac{1}{2} + \frac{p_{rf} - p_{if}}{2t}$$

$$\sigma_{rf} = \frac{1}{2} + \frac{p_{if} - p_{rf}}{2t}$$

<div style="text-align: right">(3-14)</div>

$$\pi_i^{II} = \sigma_{if}(p_{if} - c_f)$$

$$\pi_r^{II} = \sigma_{rf}(p_{rf} - c_f)$$

<div style="text-align: right">(3-15)</div>

A first order condition leads to equilibrium prices:

$$p_{if}^{II} = t + c_f$$

$$p_{rf}^{II} = t + c_f$$

<div style="text-align: right">(3-16)</div>

And profits at equilibrium:

$$\pi_i^{IN} = \frac{t}{2}$$

$$\pi_r^{IN} = \frac{t}{2}$$

<div style="text-align: right">(3-17)</div>

An increase in access charge has no effect on either firm. It is interesting to note that despite the increase in consumer utility provided by fiber, the incumbent's profits are lower than in the case where neither firm invests, while the rival's profits are the same.

	No rival investment	Rival investment
No incumbent investment	$\sigma_{ic} = \dfrac{1}{2}$ $\sigma_{rc} = \dfrac{1}{2}$	$\sigma_{ic} = \sigma_{rc} = \dfrac{1}{2} - \dfrac{3\theta - 2\omega}{6t}$ $\sigma_{if} = \sigma_{rf} = \dfrac{1}{2} + \dfrac{3\theta - 2\omega}{6t}$
Incumbent investment	$\sigma_{ic} = \sigma_{rc} = \dfrac{1}{2} + \dfrac{\omega}{6t}$ $\sigma_{if} = \sigma_{rf} = \dfrac{1}{2} - \dfrac{\omega}{6t}$	$\sigma_{if} = \dfrac{1}{2}$ $\sigma_{rf} = \dfrac{1}{2}$

Table 3 Market share table for *NN*, *NI*, *IN, II* for h=0 (FTTH-infrastructure-based competition)

3.1.5 Investment Incentives

The following payoff table summarizes the incumbent and rival's profits in four situations *NN*, *NI*, *IN, II.*

	No rival investment	Rival investment
No incumbent investment	$\pi_i^{NN} = \dfrac{t}{2} + a_c - c$ $\pi_r^{NN} = \dfrac{t}{2}$	$\pi_i^{NI} = \dfrac{(3t - \omega)^2}{18t}$ $\pi_r^{NI} = \dfrac{(3t + \omega)^2}{18t}$
Incumbent investment	$\pi_i^{IN} = a_c - c + \dfrac{(3t + \omega)^2}{18t}$ $\pi_r^{IN} = \dfrac{(3t - \omega)^2}{18t}$	$\pi_i^{IN} = \dfrac{t}{2}$ $\pi_r^{IN} = \dfrac{t}{2}$

Table 4 payoff table for *NN*, *NI*, *IN, II* for h=0 (FTTH-infrastructure-based competition)

The investment incentive is the difference between profits after investment and profits before investment.
For a given area, we assume the fixed cost of investment is the same for both firms: *f*. The denser the area, the lower the fixed FTTH cost *f*.

For a given area, incentives are: $\pi_i^{IN} - \pi_i^{NN}$ for the incumbent to invest alone; $\pi_r^{NI} - \pi_r^{NN}$ for the rival to invest alone; $\pi_i^{II} - \pi_i^{NI}$ for the incumbent to invest when the rival invests; $\pi_r^{II} - \pi_r^{IN}$ for the rival to invest when the incumbent invests.

The maximum amount that a firm is encouraged to invest corresponds to the least dense area the firm is willing to cover. By using $f_i^{\,j}$ as defined in section 2, the actions and reactions of the incumbent and its rival,

$$f_i^1 = \pi_i^{IN} - \pi_i^{NN} = \frac{\omega(6t + \omega)}{18t}$$

$$f_r^1 = \pi_r^{NI} - \pi_r^{NN} = \frac{\omega(6t + \omega)}{18t}$$

$$f_i^2 = \pi_i^{II} - \pi_i^{NI} = \frac{\omega(6t - \omega)}{18t}$$

$$f_r^2 = \pi_r^{II} - \pi_r^{IN} = \frac{\omega(6t - \omega)}{18t}$$

(3-18)

We can observe that the access charge does not appear in equation (3-18). This means that investment incentives are independent of access charge. In this case, the access charge plays no role in the investment incentives.

We can also observe that, in this case, the firms have exactly the same incentives whether they invest alone or they both invest. Finally, there are two thresholds of investment: a threshold under which both firms invest and one above which neither firm invests.
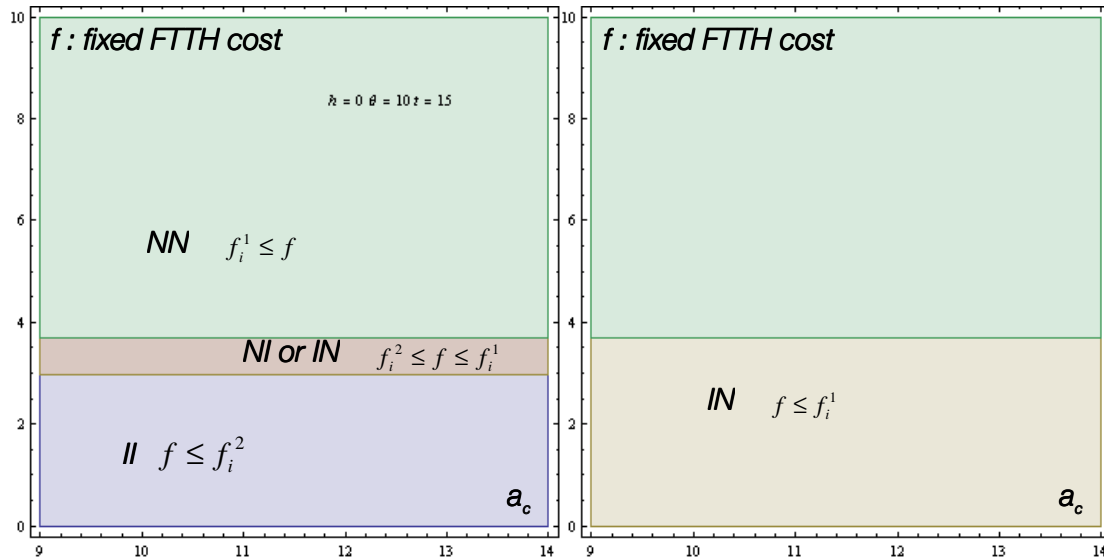

Figure 2 Investment incentives for *NN*, *NI*, *IN* and *II* as functions of a_c and f for h=0 (FTTH-infrastructure-based competition)

As a function of copper access charge and fixed FTTH cost, the lefthand region plot above indicates:

In the densest areas where $f \leq f_i^2$ both firms invest. In moderately dense areas where $f_i^2 \leq f \leq f_i^1$ only one firm invests (the incumbent or the rival). In the least dense areas where $f_i^1 \leq f$ no firm invests.

If only the incumbent has the financial capacity to invest in FTTH, the right region plot above indicates that incumbent's investment incentives are not sensitive to copper access charge.

The benefits of fiber as compared to copper, ω, play a major role. Incentives to invest (alone or both) increase with ω. The condition $\omega \leq 3h$ ensures that all incentives are positive.

Let us denote $\gamma = \dfrac{1}{t}$ which represents the intensity of competition. The thresholds become: $f_i^1 = \dfrac{\omega(6 + \gamma\omega)}{18}$ and $f_i^2 = \dfrac{\omega(6 - \gamma\omega)}{18}$. The intensity of competition therefore raises the threshold above which neither firm invests and lowers the threshold under which both firms invest. Competition tends to increase the area where only one firm invests and decrease the area where both firms invest. When there is no competition, $\gamma = 0$, in the case of two local monopolies the thresholds merge.

3.1.6 Discussion

Why do the access charge not play a role in the investment incentives?
If $h = 0$, the wholesale market disappears when the rival invests, while it is fully preserved otherwise. This is why the access charge is fully preserved in $\pi_i^{IN} = a_c - c + \dfrac{(3t + \omega)^2}{18t}$ and disappears in the difference $\pi_i^{IN} - \pi_i^{NN} = \dfrac{\omega(6t + \omega)}{18t}$
In the same manner, access charge does not appear in rival's profit expressions when the rival invests and they did not appear in π_r^{NN}. Thus the access charge does not appear in any of the incentive expressions, equation (3-18).

We can infer that this will no longer be the case if $h > 0$. In next section, we will see that the rival's incentive to invest alone increases with the access charge, while the incumbent's incentive remains steady.

What happens if regulator orders the firm which has invested alone to provide access to its competitor with a fiber access charge a_f? In next section, we will see that it will reduce the profits of the firm which has invested and thus reduce its investment incentives.

3.2 *Perfect competition t=0*

In this case, the utility functions in equation (2-1) can be rewritten:

$$
\begin{aligned}
U_{ic} &= V - p_{ic} \\
U_{if} &= V - p_{if} + \theta - h(1 - 2y) \\
U_{rc} &= V - p_{rc} \\
U_{rf} &= V - p_{rf} + \theta - h(1 - 2y)
\end{aligned}
$$

(3-19)

The equilibrium may differ depending on whether the incumbent or the rival invests or both invest.

3.2.1 No firm invests

In this case, equation (3-19) leads to $\sigma_{ic} = \sigma_{rc} = \dfrac{1}{2}$ and $p_{ic} = p_{rc} = a_c$

$$\begin{aligned} \pi_i^{NN} &= a_c - c \\ \pi_r^{NN} &= 0 \end{aligned}$$

(3-20)

The rival earns no profits and the incumbent earns the difference between the access charge and the marginal cost.

3.2.2 Only the incumbent invests

In this case, equation (3-19) and perfect competition lead to $p_{ic} = p_{rc} = a_c$, and $\sigma_{ic} = \sigma_{rc} = \dfrac{1-\sigma_{if}}{2}$ with $\sigma_{if} = -\dfrac{p_{if} - a_c - \theta - h}{2h}$.

Profits at equilibrium are written:

$$\begin{aligned} \pi_i^{IN} &= \sigma_{ic}(a_c - c) + \sigma_{rc}(a_c - c) + \sigma_{if}(p_{rf} - c_f) \\ \pi_r^{IN} &= 0 \end{aligned}$$

(3-21)

A first order condition leads to:

$$p_{if} = c_f + a_c - c + \frac{h + \omega}{2}$$

(3-22)

We assume that $\omega \le 3h$ in order to ensure that the copper market share is positive.

And profits at equilibrium are written:

$$\begin{aligned} \pi_i^{IN} &= a_c - c + \frac{(h + \omega)^2}{8h} \\ \pi_r^{IN} &= 0 \end{aligned}$$

(3-23)

In this case, fiber enables the incumbent to increase its profits. As only the incumbent has invested, it has the monopoly on fiber, which decreases competition. The incumbent can set its fiber price above marginal cost and thus increase its profits on fiber while maintaining the profits provided by access charges on copper. It is interesting to note that the incumbent maintains the profit provided by access charges not only for copper but also for fiber, because of the expression $(a_c - c)$ for its fiber price. The incumbent thus fully maintains its profits generated by access charges and can increase its profits from copper.
The rival does not benefit from fiber and cannot increase its profits.

3.2.3 Only the rival invests

In this case, equation (3-19) and perfect competition again lead to $p_{ic} = p_{rc} = a_c$, and

$$\sigma_{ic} = \sigma_{rc} = \frac{1 - \sigma_{rf}}{2} \text{ with } \sigma_{rf} = \frac{p_{if} - a_c - \theta - h}{2h}.$$

Remark: $p_{ic} > p_{rc}$ or $p_{ic} < p_{rc}$ does not lead to Nash equilibrium.

Profits are written:

$$\pi_i^{NI} = \sigma_{ic}(a_c - c) + \sigma_{rc}(a_c - c)$$
$$\pi_r^{NI} = \sigma_{rf}(p_{rf} - c_f)$$

(3-24)

A first order condition leads to:

$$p_{rf} = c_f + \frac{a_c - c + h + \omega}{2}$$

(3-25)

We assume that $\omega \leq 3h - (a_c - c)$ in order to ensure that the copper market share is positive.

Profits at equilibrium are written:

$$\pi_i^{NI} = \frac{(a_c - c)(3h - \omega - (a_c - c))}{4h}$$
$$\pi_r^{NI} = \frac{(a_c - c + h + \omega)^2}{8h}$$

(3-26)

Only the rival has invested, so it has a monopoly on fiber , which alleviates competition. The rival can set its fiber price above marginal cost and thus increase its profits on fiber. Its profits on copper remain nil. The copper market share decreases, reducing the incumbent's profits generated by access charges. It is relevant to note that the rival has captured part of the profits generated by the access charges.

3.2.4 Both firms invest

In this case, perfect competition requires that the firm set prices at marginal cost for fiber and copper access charges, as in subsection 3.2.1.
Since both firms have invested, there is also perfect competition on fiber, which no longer alleviates competition as in subsections 3.2.2 and 3.2.3.
A first order condition thus leads to equilibrium prices $p_{ic} = p_{rc} = a_c$ and $p_{if} = p_{rf} = c_f$.
Profits at equilibrium are written:

$$\pi_i^{II} = \frac{(a_c - c)(h - \omega - (a_c - c))}{2h}$$
$$\pi_r^{II} = 0$$

(3-27)

The rival earns no profits. The access charge generate profits for the incumbent, which are not only proportional to the difference between the access charge and the marginal cost of copper marginal $(a_c - c)$, as in subsection 3.2.1, but are also proportional to the copper market share. In this case, however, the introduction of fiber reduces the copper market share, thus reducing the incumbent's profit as compared to when neither firm invests (subsection 3.2.1).

3.2.5 Investment incentives

The following payoff table summarizes the incumbent and rival's profits in four situations *NN*, *NI*, *IN, II.*

	No rival investment	Rival investment
No incumbent investment	$\pi_i^{NN} = a_c - c$ $\pi_r^{NN} = 0$	$\pi_i^{NI} = \dfrac{(a_c - c)(3h - \omega - (a_c - c))}{4h}$ $\pi_r^{NI} = \dfrac{(a_c - c + h + \omega)^2}{8h}$
Incumbent investment	$\pi_i^{IN} = a_c - c + \dfrac{(h + \omega)^2}{8h}$ $\pi_r^{IN} = 0$	$\pi_i^{II} = \dfrac{(a_c - c)(h - \omega - (a_c - c))}{2h}$ $\pi_r^{II} = 0$

Table 5 Payoff table for *NN, NI, IN, II* for t=0 (FTTH-infrastructure-based competition)

As in subsection 3.1.5, the investment incentives for a given area are:

$$f_i^1 = \pi_i^{IN} - \pi_i^{NN} = \frac{(h + \omega)^2}{8h}$$

$$f_r^1 = \pi_r^{NI} - \pi_r^{NN} = \frac{(a_c - c + h + \omega)^2}{8h}$$

$$f_i^2 = \pi_i^{II} - \pi_i^{NI} = -\frac{(a_c - c)(a_c - c + h + \omega)}{4h}$$

$$f_r^2 = \pi_r^{II} - \pi_r^{IN} = 0$$

In the densest areas where $f \le f_i^2$ both firms invest

In moderately dense areas where $f_i^2 \le f \le f_i^1$ only one firm invests (the incumbent or the rival)

In the less dense areas where $f_i^1 \le f$ no firm invests

When $t = 0$, as opposed to when $h = 0$, access charge plays a role in the investment incentives.

Access charges increase the rival's incentive to invest alone. It should be noted that when the access charge is greater than the copper marginal cost, $a_c > c$, the

incumbent's incentive to invest when the rival invests f_i^2 is negative. This means that the incumbent is better off when the rival invests alone than when both firms invest. The market share of copper is larger when the rival invests alone than when both firms invest, and in both cases the incumbent's profits are generated by the access charges and thus depend on the market share of copper.

The incumbent's investment incentives alone do not depend on the access charge because, as we saw in subsection 3.2.2, the incumbent maintains its part of the profits they generate.

The area where neither firm invests thus decrease with the access charge, while the area where only the rival invests increases with the access charge and there is no area where both firms invest because neither the incumbent nor the rival is interested in investing when their competitor invests.
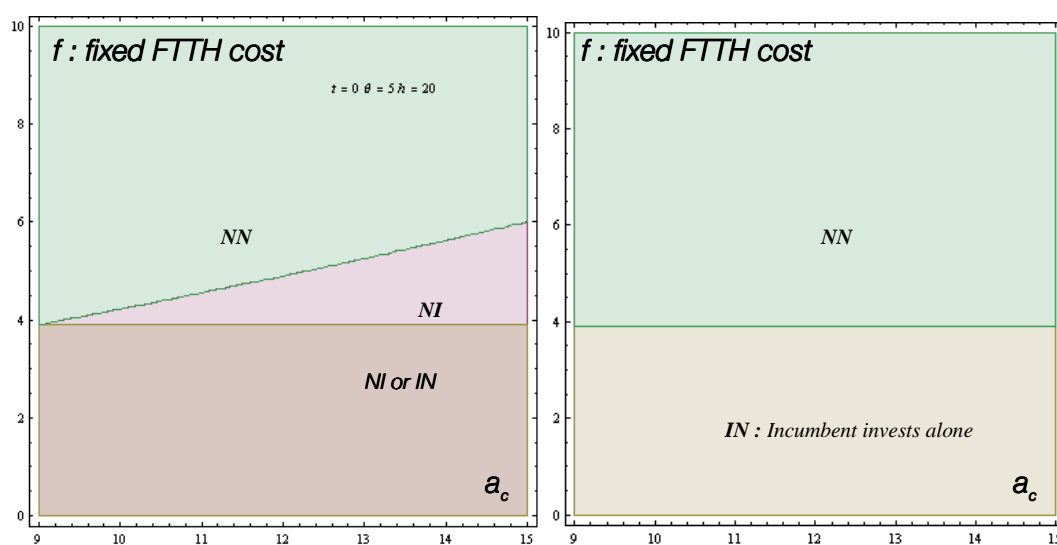

Figure 3 Investment incentives for *NN*, *NI*, *IN* and *II* as functions of a_c and *f* for t=0 (FTTH-infrastructure-based competition)

The lefthand region plot above represents the areas where neither firm invests (*NN*), only Incumbent invests (*IN*), only Rival invests (*IN*) or both firms invest (*II*) as a function of the copper access charge and fixed FTTH costs.

When only the incumbent has the financial capacity to invest in FTTH, the righthand region plot above shows that the incumbent's investment incentives are not sensitive to the copper access charge.

The marginal social surplus generated by fiber, represented by the parameter ω, increases fiber coverage. The vertical differentiation parameter, *h*, tends to increase coverage when $\frac{\partial f_r^1}{\partial h} > 0$. This is the case when $h > a_c - c - \omega$. Otherwise, *h* tends to decrease the coverage.

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3.3 Imperfect competition in a segmented market t>0 h>0

In this section, the four situations *NN, NI, IN, II* are again analyzed with *t > 0 and h > 0*. *t > 0* means that we are dealing with imperfect competition. *h > 0* indicates that consumers are segmented: some consumers prefer copper access and others prefer fiber access. The two types of access are not totally substitutable.

3.3.1 Equilibrium price table for *NN, NI, IN, II*

In this subsection, equilibrium prices for *NN, NI, IN, II* are summarized in the table below as a function of copper access charge with `c=9;`θ`=5;h=20;t=15;c`$_f$`=9;`
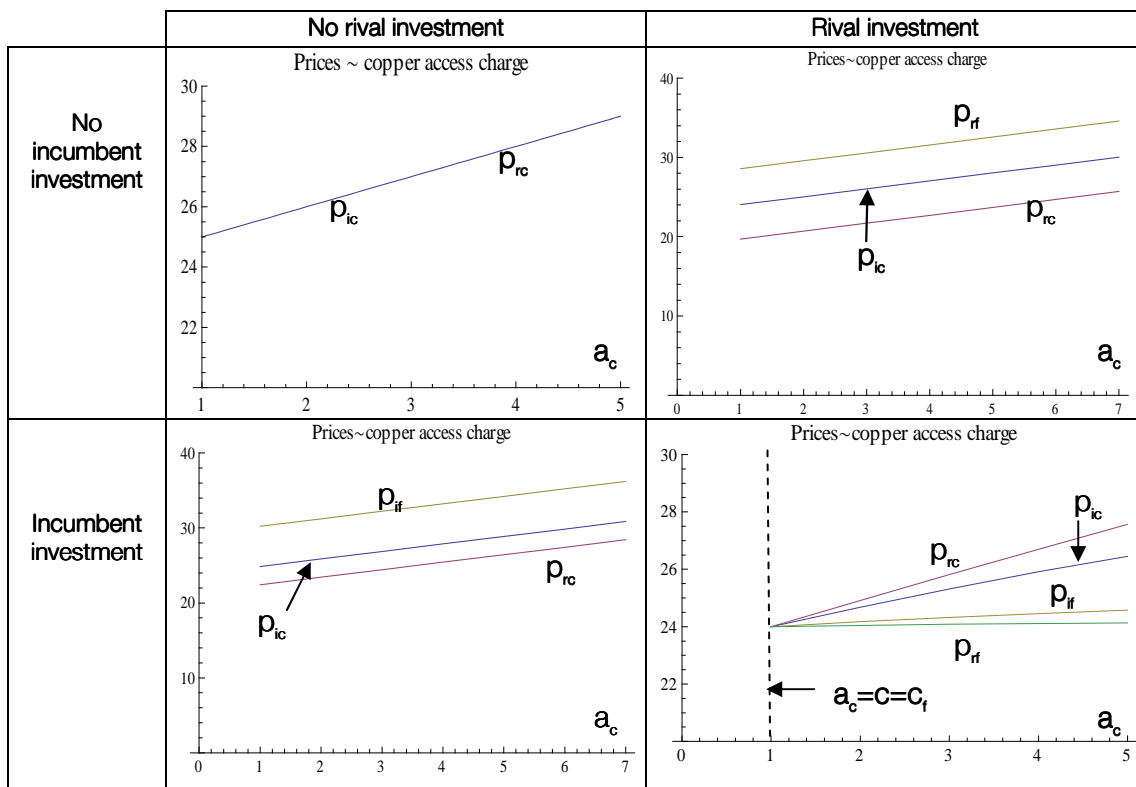
Figure 4 Equilibrium prices *NN, NI, IN* and *II* as a function of a_c for h > 0 t > 0 (FTTH-infrastructure-based competition)

In situation *NN*, the equilibrium prices (p_{ic}, p_{rc}) both increase with a_c.

In situation *NI*, the equilibrium price curves above indicate that the rival's FTTH price is higher than both copper prices due to consumers' increased willingness to pay for FTTH. The rival's copper price is lower than the incumbent's in order to maintain consumers' demand. All prices increase with the access charge.

In situation *IN*, the equilibrium price curves indicate that the incumbent's FTTH price is higher than both copper prices due to consumers' increased willingness to pay for FTTH. The rival's copper price is lower than the incumbent's in order to maintain consumers' demand. All prices increase with the access charge.

In situation *II*, the equilibrium price curves indicate that all prices increase with the copper access charge. When $a_c=c=c_f$, FTTH prices are equal to copper prices. The FTTH prices do not reflect consumers' higher willingness to pay for FTTH, which latter is offset by competition between the two firms, which have both invested in FTTH.

3.3.2 Market share table at equilibrium for *NN, NI, IN, II*

In this subsection, the market shares at equilibrium for *NN, NI, IN, II* are summarized in the figure below
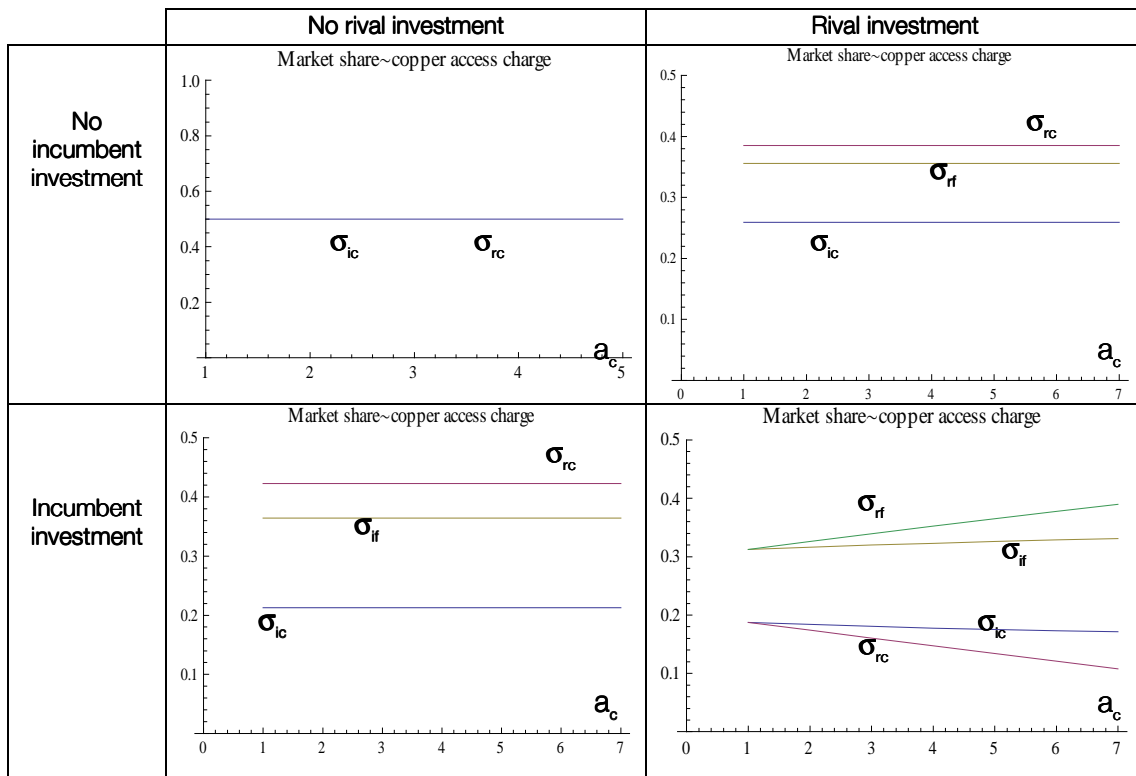

Figure 5 Market shares at equilibrium *NN, NI, IN* and *II* as a function of a_c for h > 0 t > 0 (FTTH-infrastructure-based competition)

In situation *NN*, the market shares of both copper offers at equilibrium are equal to 1/2.

In situation *NI*, the market share curves above show that the market share of each offer is stable when access charge increases. The rival's copper market share is the largest.

In situation *IN*, the market share curves above show that each offer's market share is stable when access charge increases. The rival's copper market share is the largest.

In situation *II*, the market share curves show that the rival's FTTH market share increases with access charge. The market share of the rival's copper offer decreases with access charge. A higher access charge encourages the rival's consumers to migrate from copper to FTTH. The higher willingness to pay for FTTH is expressed here by a higher market share for FTTH.

3.3.3 Profit table at equilibrium for *NN, NI, IN, II*

In this subsection, profits of each offer at equilibrium for *NN, NI, IN, II* are summarized in the table below
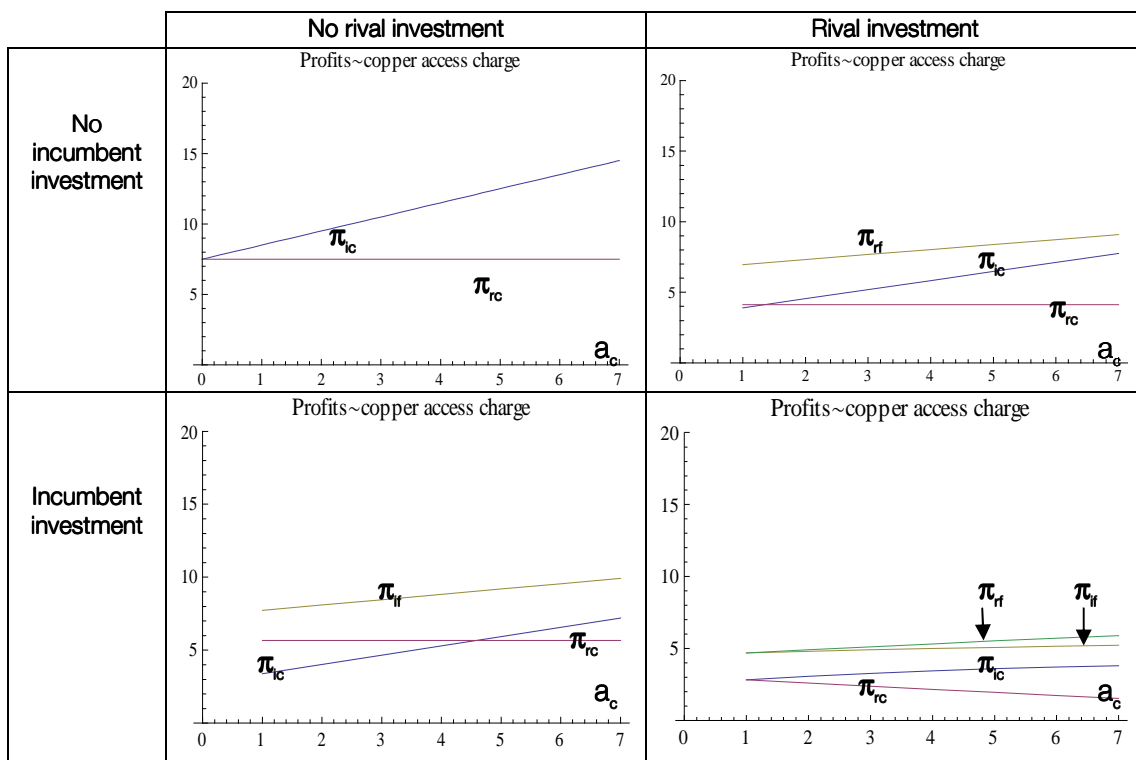

Figure 6 Profits at equilibrium *NN*, *NI*, *IN* and *II* as a function of a_c for h > 0 t > 0 (FTTH-infrastructure-based competition)

In situation *NN*, the incumbent's profits increase with the copper access charge while the rival's profit is not sensitive to a_c.

In situation *NI*, the rival's FTTH profits increase with the access charge, while the rival's copper profits are stable. Unlike the situation where t = 0 when copper offers are set at its marginal cost, the rival takes advantage of its monopoly on fiber. The prices and profits for the rival's fiber are therefore high.

In situation *IN*, the incumbent's FTTH profits increase with the access charge. The rival's profits, however, are stable. Unlike the situation where t = 0 when copper offers are set at marginal cost, the incumbent takes advantage of its monopoly on fiber. The prices and profits for the incumbent's fiber are therefore high.

In situation *II,* the incumbent's copper profits and the incumbent and rival's FTTH profits increase with the access charge. However, the rival's copper profits decrease with the access charge.

3.3.4 Nash equilibrium to determine FTTH investment incentives

In this subsection, the results of the previous subsection are summarized in a payoff table for each firm (instead of each offer) in order to determine each firm's dominant investment strategy. The sum of the profits for each firm (copper + FTTH) is shown on the curves below for *f*=0.
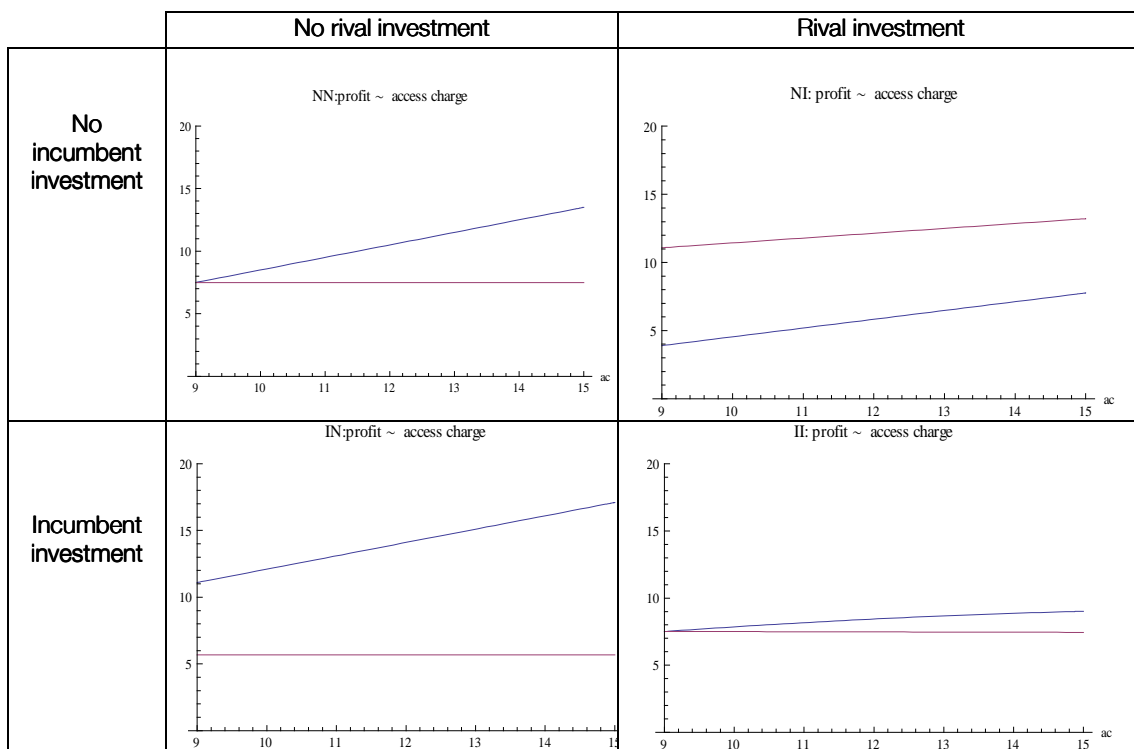
	No rival investment	Rival investment
No incumbent investment		
Incumbent investment		

Figure 7 Profits at equilibrium *NN, NI, IN* and *II* as a function of a_c for h > 0 t > 0 (FTTH-infrastructure-based competition, *f=0*)

It is clear that the two situations which correspond to "Incumbent invests in FTTH and Rival does not ("*IN*") an "the incumbent does not invest in FTTH and the rival does ("*NI*") are not dominant investment strategies for *f=0*. One of the two players sees its profit fall when its competitor invests in FTTH. The "*II*" situation where the incumbent and its rival both invest in FTTH, is the dominant

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investment strategy in this game: both see their profits increase provided that the fixed FTTH deployment cost is not too high. In other words, the reasoning above is valid for areas where FTTH deployment is naturally profitable. An equilibrium with or without FTTH investment mainly depends on two parameters: the fixed FTTH deployment cost and copper access charge.

The diagrams below show the regions *NN, NI, IN, II* for *c = 9€*/month (a typical value in Europe):
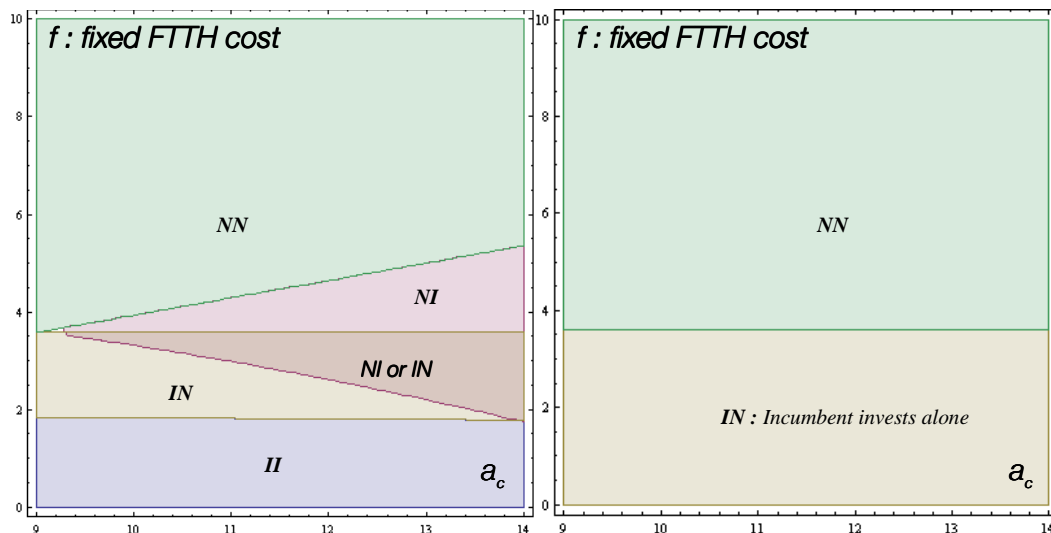

Figure 8 Investment incentives for *NN*, *NI*, *IN* and *II* as functions of a_c and *f* for h > 0 t > 0 (FTTH-infrastructure-based competition)

The lefthand region plot above indicates the following as a function of copper access charge and fixed FTTH cost:

- "*II*" is located in low *f* areas and is almost independent of a_c: when *f* is low enough, both firms invest.
- "*IN*" is located in higher *f* areas and is almost independent of a_c: The incumbent invests in FTTH only if FTTH is exclusively reserved for its use.
- "*NI*" is located in higher *f* areas and increases with a_c. A higher copper access charge encourages the rival to invest in FTTH
- "*NN*" is located in the highest *f* areas: when *f* is too high, nobody invests in FTTH

When only the incumbent has the financial capacity to invest in FTTH, the righthand region plot above shows that the incumbent's investment incentives are not sensitive to copper access charge.

4 Model resolution in FTTH-service-based competition

When the competition is FTTH-service based, four offers exist on the broadband market the "*NI*", "*IN*" and "*II*" situations. Only "*NN*" always has two offers due to the lack of an FTTH network.

Two cases are studied in this section. Subsection 4.1 analyses the investment incentives in a perfect competition where $t=0$. Subsection 4.2 analyses investment incentives in an imperfect competition within a segmented market ($t > 0$ and $h > 0$). We will focus our study on $a_c \sim c$, i.e. the level of copper access charge are near the marginal cost and $a_f \sim c_f$ i.e. the level of FTTH access charge are near the marginal cost.

4.1 Perfect competition t=0 in FTTH-service-based competition

4.1.1 Price table for *NN, NI, IN, II* with t=0

	No rival investment	Rival investment
No incumbent investment	$p_{ic} = a_c$ $p_{rc} = a_c$	$p_{ic} = a_c$ $p_{rc} = a_c$ $p_{if} = a_f$ $p_{rf} = a_f$
Incumbent investment	$p_{ic} = a_c$ $p_{rc} = a_c$ $p_{if} = a_f$ $p_{rf} = a_f$	$p_{ic} = a_c$ $p_{rc} = a_c$ $p_{if} = c_f$ $p_{rf} = c_f$

Table 6 Equilibrium price table for *NN, NI, IN, II* at *t*=0 (FTTH-service-based competition)

A perfect competition leads to setting all prices at marginal cost. Network owners' prices are aligned with their competitors'. Network owners' prices are thus equal to the access charge paid by their competitors (here a_c or a_f for copper and fiber network owners respectively).

4.1.2 Market share table for *NN, NI, IN, II* with t=0

	No rival investment	Rival investment
No incumbent investment	$\sigma_{ic} = \dfrac{1}{2}$ $\sigma_{rc} = \dfrac{1}{2}$	$\sigma_{ic} = \sigma_{rc} = \dfrac{a_f - a_c + h - \vartheta}{4h}$ $\sigma_{if} = \sigma_{rf} = \dfrac{1}{2} - \dfrac{a_f - a_c + h - \vartheta}{4h}$
Incumbent investment	$\sigma_{ic} = \sigma_{rc} = \dfrac{a_f - a_c + h - \vartheta}{4h}$ $\sigma_{if} = \sigma_{rf} = \dfrac{1}{2} - \dfrac{a_f - a_c + h - \vartheta}{4h}$	$\sigma_{ic} = \sigma_{rc} = \dfrac{c_f - a_c + h - \vartheta}{4h}$ $\sigma_{if} = \sigma_{rf} = \dfrac{1}{2} - \dfrac{c_f - a_c + h - \vartheta}{4h}$

Table 7 Market share at equilibrium for *NN, NI, IN, II* at *t*=0 (FTTH-service-based competition)

The market shares of copper offers increase with a_f and decrease with a_c. Inversely, market shares of FTTH offers increase with a_c and decrease with a_f. We can thus conclude that high copper access charge favors migration from copper to fiber.

4.1.3 Payoff table for *NN, NI, IN, II* with t=0

	No rival investment	Rival investment
No incumbent investment	$\pi_i^{NN} = a_c - c$ $\pi_r^{NN} = 0$	$\pi_i^{NI} = \dfrac{(a_c - c)(h - \vartheta + (a_f - a_c))}{2h}$ $\pi_r^{NI} = (a_f - c_f)(1 - \dfrac{(h - \vartheta + (a_f - a_c))}{2h})$
Incumbent investment	$\pi_i^{IN} = \dfrac{(a_c - c)(h - \vartheta + (a_f - a_c))}{2h}$ $+ (a_f - c_f)(1 - \dfrac{(h - \vartheta + (a_f - a_c))}{2h})$ $\pi_r^{IN} = 0$	$\pi_i^{II} = \dfrac{(a_c - c)(h - \vartheta + (c_f - a_c))}{2h}$ $\pi_r^{II} = 0$

Table 8 Profits at equilibrium for *NN, NI, IN, II* at *t*=0 (FTTH-service-based competition)

4.1.4 Investment incentives t=0

In this subsection, the results of the payoff table are used to determine the dominant investment strategy for both firms.
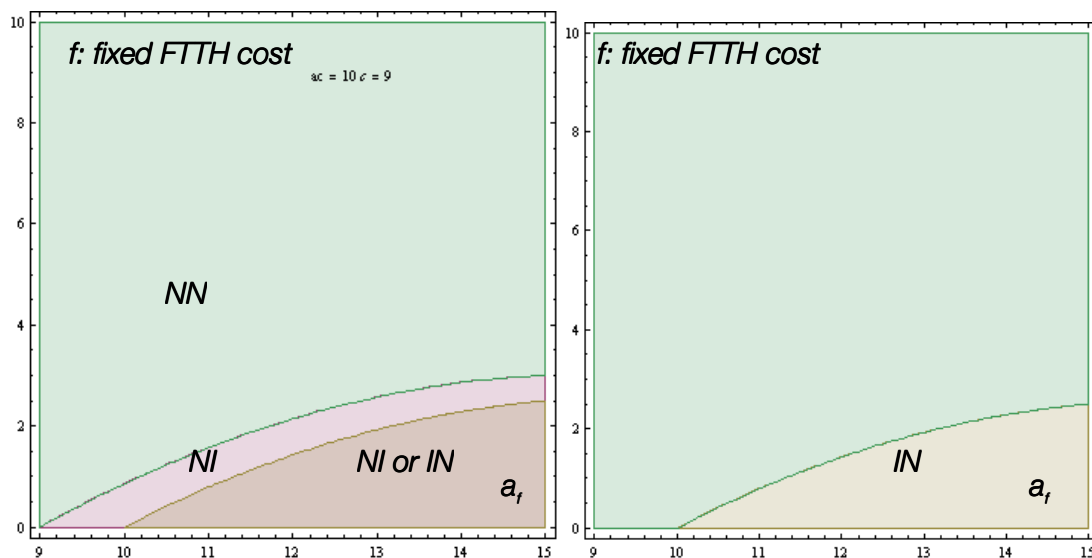

Figure 9 Investment incentives for *NN, NI, IN* and *II* as functions of a_c and *f* for t=0

The lefthand region plot above indicates the following as a function of FTTH access charge and fixed FTTH cost:

- The "*II*" region is absent. A perfect competition cancels the incentives for a both firms to invest simultaneously.
- The "*IN*" region increases with FTTH access charge a_f.
- The "*NI*" region also increases with the FTTH access charge a_f. A higher copper access charge ($a_c > c$) encourages the rival to invest more in FTTH than the incumbent.
- The "*NN*" region occurs for higher FTTH fixed cost *f*: when *f* is too high, neither firm invests in FTTH.

If only the incumbent has the financial capacity to invest in FTTH, the righthand region plot above indicates that the incumbent's investment incentives increase with the FTTH access charge.

4.2 Imperfect competition *t* > 0 and h > 0 at $a_c \sim c$, $a_f \sim c_f$ in FTTH-service-based competition

The previous subsection shows that investment incentives for both firms increase with the FTTH access charge in a perfect competition. We will now examine a generic case where t > 0 and h > 0. In this subsection, situations "*II*", "*NI*" and "*IN*", will be studied with the implicit function and Taylor series. We can find the behaviors of equilibrium prices, the market shares and the profits for each offer at $a_c \sim c$ and $a_f \sim c_r$. The objective is to confirm the investment incentives with respect to a_f and a_c.

4.2.1 Price table and derivatives with respect to a_c and a_f for *NN*, *NI*, *IN*, *II*

The equilibrium prices are calculated using the first order condition at $a_f = c_f$ and $a_c = c$:

	No rival investment	Rival investment
No incumbent investment	$p_{ic} = t + c$ $p_{rc} = t + c$	$p_{ic} = t + c$ $p_{rc} = t + c$ $p_{if} = t + c_f$ $p_{rf} = t + c_f$
Incumbent investment	$p_{ic} = t + c$ $p_{rc} = t + c$ $p_{if} = t + c_f$ $p_{rf} = t + c_f$	$p_{ic} = t + c$ $p_{rc} = t + c$ $p_{if} = t + c_f$ $p_{rf} = t + c_f$

Table 9 Equilibrium price table for *NN, NI, IN, II* at *t>0 ,h>0, a_c=c, a_f=c_f* (FTTH-service-based competition)

The derivative of equilibrium prices is calculated using implicit functions with respect to a_c and a_f at $a_f=c_f$ and $a_c=c$.

The first order condition to obtain the equilibrium prices is $\pi'(p^*(a),a)=0$ where the equilibrium price p* is a function of access charge. The implicit function gives

$$\frac{dp^*}{da}=-\frac{\frac{\partial \pi'}{\partial a}}{\frac{\partial \pi'}{\partial p^*}} \tag{4-1}$$

By extending equation (4-1) with $\pi'=\left\{\dfrac{d\pi_i}{dp_{ic}},\dfrac{d\pi_i}{dp_{if}},\dfrac{d\pi_r}{dp_{rc}},\dfrac{d\pi_r}{dp_{rf}}\right\}$ and $a=\left\{a_c,a_f\right\}$, we obtain

	No rival investment	Rival investment
No incumbent investment	$\dfrac{dp_{ic}}{da_c}=1$ $\dfrac{dp_{rc}}{da_c}=1$	$\dfrac{dp_{ic}}{da_f}=\dfrac{2t(h+\omega)(h(h+t)-\omega^2)}{(3h^2+2ht-3\omega^2)(h^2+2ht-\omega^2)}$ $\dfrac{dp_{rc}}{da_f}=\dfrac{t(h-\omega)(h+\omega)^2}{(3h^2+2ht-3\omega^2)(h^2+2ht-\omega^2)}$ $\dfrac{dp_{if}}{da_f}=\dfrac{1}{2}(2+\dfrac{t(-h+\omega)}{3h^2+2ht-3\omega^2}+\dfrac{t(-h+\omega)}{h^2+2ht-\omega^2})$ $\dfrac{dp_{rf}}{da_f}=\dfrac{1}{2}(2-\dfrac{2t(h-\omega)^2(h+\omega)}{(3h^2+2ht-3\omega^2)(h^2+2ht-\omega^2)})$
Incumbent investment	$\dfrac{dp_{ic}}{da_f}=\dfrac{2t(h+\omega)(h(h+t)-\omega^2)}{(3h^2+2ht-3\omega^2)(h^2+2ht-\omega^2)}$ $\dfrac{dp_{rc}}{da_f}=\dfrac{t(h-\omega)(h+\omega)^2}{(3h^2+2ht-3\omega^2)(h^2+2ht-\omega^2)}$ $\dfrac{dp_{if}}{da_f}=\dfrac{1}{2}(2+\dfrac{t(-h+\omega)}{3h^2+2ht-3\omega^2}+\dfrac{t(-h+\omega)}{h^2+2ht-\omega^2})$ $\dfrac{dp_{rf}}{da_f}=\dfrac{1}{2}(2-\dfrac{2t(h-\omega)^2(h+\omega)}{(3h^2+2ht-3\omega^2)(h^2+2ht-\omega^2)})$	$\dfrac{dp_{ic}}{da_c}=\dfrac{1}{2}(2-\dfrac{t(h+\omega)}{3h^2+2ht-3\omega^2}-\dfrac{t(h+\omega)}{h^2+2ht-\omega^2})$ $\dfrac{dp_{rc}}{da_c}=\dfrac{1}{2}(2+\dfrac{t(h+\omega)}{3h^2+2ht-3\omega^2}-\dfrac{t(h+\omega)}{h^2+2ht-\omega^2})$ $\dfrac{dp_{if}}{da_c}=\dfrac{2t(h-\omega)(h(h+t)-\omega^2)}{(3h^2+2ht-3\omega^2)(h^2+2ht-\omega^2)}$ $\dfrac{dp_{rf}}{da_c}=\dfrac{t(h-\omega)^2(h+\omega)}{(3h^2+2ht-3\omega^2)(h^2+2ht-\omega^2)}$

Table 10 Derivatives of equilibrium prices at *t>0, h>0, a_c=c, a_f=c_f* (FTTH-service-based competition)

It can be demonstrated that $\dfrac{dp_{ic}}{da_c}$, $\dfrac{dp_{rc}}{da_c}$, $\dfrac{dp_{if}}{da_c}$ and $\dfrac{dp_{rf}}{da_c}$ are both positive if $t>0$ and $h>\omega$. In the next subsection, we will see that $h>\omega$ is necessary to have a positive copper market share. These positive expressions indicate that an increase in the copper access charge leads to an increase in both prices.

It can also be demonstrated that $\dfrac{dp_{ic}}{da_f}$ and $\dfrac{dp_{rc}}{da_f}$ are always negative, $\dfrac{dp_{if}}{da_f}$ and $\dfrac{dp_{rf}}{da_f}$ are always positive if $t > 0$ and $h > \omega$. These expressions indicate that an increase in FTTH access charge leads to a decrease in copper prices and an increase in FTTH prices.

4.2.2 Market share table and derivatives with respect to a_c and a_f for *NN,NI, IN, II*

The market share of each offer is calculated using the equilibrium prices at $a_f=c_f$ and $a_c=c$

	No rival investment	Rival investment
No incumbent investment	$\sigma_{ic} = \dfrac{1}{2}$ $\sigma_{rc} = \dfrac{1}{2}$	$\sigma_{ic} = \sigma_{rc} = \dfrac{h-\omega}{4h}$ $\sigma_{if} = \sigma_{rf} = \dfrac{h+\omega}{4h}$
Incumbent investment	$\sigma_{ic} = \sigma_{rc} = \dfrac{h-\omega}{4h}$ $\sigma_{if} = \sigma_{rf} = \dfrac{h+\omega}{4h}$	$\sigma_{ic} = \sigma_{rc} = \dfrac{h-\omega}{4h}$ $\sigma_{if} = \sigma_{rf} = \dfrac{h+\omega}{4h}$

Table 11 Market share at equilibrium at *t>0 , h>0, a_c=c, a_f=c_f* (FTTH-service-based competition)

The table above shows that in situations *NI*, *IN* and *II*, the copper market share is positive if and only if $h > \omega$. Consumers should be sufficiently segmented for the copper market not to be empty.

Since market share is a function of prices, using the results obtained above for equilibrium prices and their derivatives at $a_c=c$ and $a_f=c_f$, the market share derivative of each offer can be calculated with respect to a_c and a_f at $a_f=c_f$ and $a_c=c$:

	No rival investment	Rival investment
No incumbent investment	$$\frac{d\sigma_{ic}}{da_c} = 0$$ $$\frac{d\sigma_{rc}}{da_c} = 0$$	$$\frac{d\sigma_{ic}}{da_f} = \frac{1}{12}\left(\frac{2}{h} + \frac{-3t}{h^2+2ht-\omega^2} + \frac{t}{-3h^2-2ht+3\omega^2}\right)$$ $$\frac{d\sigma_{rc}}{da_f} = \frac{(-h(h+t)+\omega^2)^2}{h(3h^2+2ht-3\omega^2)(h^2+2ht-\omega^2)}$$ $$\frac{d\sigma_{if}}{da_f} = \frac{1}{12}\left(-\frac{2}{h} + \frac{t}{3h^2+2ht-3\omega^2} + \frac{3t}{h^2+2ht-\omega^2}\right)$$ $$\frac{d\sigma_{rf}}{da_f} = -\frac{(-h(h+t)+\omega^2)^2}{h(3h^2+2ht-3\omega^2)(h^2+2ht-\omega^2)}$$
Incumbent investment	$$\frac{d\sigma_{ic}}{da_f} = \frac{1}{12}\left(\frac{2}{h} + \frac{-3t}{h^2+2ht-\omega^2} + \frac{t}{-3h^2-2ht+3\omega^2}\right)$$ $$\frac{d\sigma_{rc}}{da_f} = \frac{(-h(h+t)+\omega^2)^2}{h(3h^2+2ht-3\omega^2)(h^2+2ht-\omega^2)}$$ $$\frac{d\sigma_{if}}{da_f} = \frac{1}{12}\left(-\frac{2}{h} + \frac{t}{3h^2+2ht-3\omega^2} + \frac{3t}{h^2+2ht-\omega^2}\right)$$ $$\frac{d\sigma_{rf}}{da_f} = -\frac{(-h(h+t)+\omega^2)^2}{h(3h^2+2ht-3\omega^2)(h^2+2ht-\omega^2)}$$	$$\frac{d\sigma_{ic}}{da_c} = \frac{1}{12}\left(\frac{2}{h} + \frac{-3t}{h^2+2ht-\omega^2} + \frac{t}{-3h^2-2ht+3\omega^2}\right)$$ $$\frac{d\sigma_{rc}}{da_c} = \frac{(-h(h+t)+\omega^2)^2}{h(3h^2+2ht-3\omega^2)(h^2+2ht-\omega^2)}$$ $$\frac{d\sigma_{if}}{da_c} = \frac{1}{12}\left(-\frac{2}{h} + \frac{t}{3h^2+2ht-3\omega^2} + \frac{3t}{h^2+2ht-\omega^2}\right)$$ $$\frac{d\sigma_{rf}}{da_c} = -\frac{(-h(h+t)+\omega^2)^2}{h(3h^2+2ht-3\omega^2)(h^2+2ht-\omega^2)}$$

Table 12 Market share derivatives at equilibrium at $t>0$, $h>0$, $a_c=c$, $a_f=c_f$ (FTTH-service-based competition)

It can be demonstrated that $\dfrac{d\sigma_{if}}{da_c}$ and $\dfrac{d\sigma_{rf}}{da_c}$ are always positive if $t > \dfrac{\omega^2-h^2}{2h}$.

Since $h > \omega$, so $\dfrac{\omega^2-h^2}{2h} < 0$. Consequently, $t > 0 > \dfrac{\omega^2-h^2}{2h}$ is always true. These positive expressions indicate that an increase in copper access charge favors FTTH market shares. It can also be demonstrated that $\dfrac{d\sigma_{ic}}{da_f}$ and $\dfrac{d\sigma_{rc}}{da_f}$ are always positive

and $\dfrac{d\sigma_{if}}{da_f}$ and $\dfrac{d\sigma_{rf}}{da_f}$ are always negative. This means that an increase in FTTH access charge boosts copper market shares to the detriment of FTTH market shares.

4.2.3 Payoff table and derivatives with respect to a_c and a_f for *NN, NI, IN, II*

The payoff of each offer is calculated using the equilibrium prices at $a_f=c_f$ and $a_c=c$:

	No rival investment	Rival investment
No incumbent investment	$\pi_i^{NN} = \dfrac{t}{2}$ $\pi_r^{NN} = \dfrac{t}{2}$	$\pi_i^{NI} = \dfrac{t}{2}$ $\pi_r^{NI} = \dfrac{t}{2} - f$
Incumbent investment	$\pi_i^{IN} = \dfrac{t}{2} - f$ $\pi_r^{IN} = \dfrac{t}{2}$	$\pi_i^{II} = \dfrac{t}{2} - f$ $\pi_r^{II} = \dfrac{t}{2} - f$

Table 13 Profits at equilibrium at *t>0 ,h>0, a_c=c, a_f=c_f* (FTTH-service-based competition)

Since profits are also function of prices, using the previous results obtained for equilibrium prices and their derivatives at $a_c=c$ and $a_f=c_f$, the profit derivatives of each offer can be calculated with respect to a_c and a_f at $a_f=c_f$ and $a_c=c$:

	No rival investment	Rival investment	
No incumbent investment	$\dfrac{d\pi_i^*}{da_c} = 1$ $\dfrac{d\pi_r^*}{da_c} = 0$	$\dfrac{d\pi_i^*}{da_f} = 0$ $\dfrac{d\pi_r^*}{da_f} = \dfrac{h+\omega}{2h}$	$\dfrac{d\pi_i^*}{da_c} = \dfrac{h-\omega}{2h}$ $\dfrac{d\pi_r^*}{da_c} = 0$
Incumbent investment	$\dfrac{d\pi_i^*}{da_f} = \dfrac{h+\omega}{2h}$ $\dfrac{d\pi_i^*}{da_c} = \dfrac{h-\omega}{2h}$ $\dfrac{d\pi_r^*}{da_f} = 0$ $\dfrac{d\pi_r^*}{da_c} = 0$	$\dfrac{d\pi_i^*}{da_c} = \dfrac{h-\omega}{2h}$ $\dfrac{d\pi_r^*}{da_c} = 0$	

Table 14 Derivatives of profits at *t > 0, h > 0, a_c=c, a_f=c_f* (FTTH-service-based competition)

It can be demonstrated that $\dfrac{d\pi_i}{da_c}$ is always positive if $h > \omega$, a required condition for the copper market share, remains positive. This expression indicates that an increase in copper access charge leads to an increase in the incumbent's profit and does not impact the rival's profit. However, since $\dfrac{h+\omega}{2h}$ is always positive with $h > 0$ and $\omega > 0$, an increase in FTTH access charge leads to an increase in profits for the firm which invested in FTTH.

4.2.4 Investment incentives

In this subsection, the results of the payoff table are used to determine the dominant investment strategy for both firms.

By combining two previous tables and using a Taylor series up to the first order, i.e.

$$\pi = \pi\Big|_{a=c} + \frac{d\pi}{da}\Big|_{a=c} (a-c) \, ,$$ we can obtain the profits table for a_c~c and a_f~c_f as follow

	No rival investment	rival investment
No incumbent investment	$\pi_i^{NN} = \dfrac{t}{2} + a_c - c$ $\pi_r^{NN} = \dfrac{t}{2}$	$\pi_i^{NI} = \dfrac{t}{2} + \dfrac{h-\omega}{2h}(a_c - c)$ $\pi_r^{NI} = \dfrac{t}{2} + \dfrac{h+\omega}{2h}(a_f - c_f) - f$
incumbent investment	$\pi_i^{IN} = \dfrac{t}{2} + \dfrac{h+\omega}{2h}(a_f - c_f) + \dfrac{h-\omega}{2h}(a_c - c) - f$ $\pi_r^{IN} = \dfrac{t}{2}$	$\pi_i^{II} = \dfrac{t}{2} + \dfrac{h-\omega}{2h}(a_c - c) - f$ $\pi_r^{II} = \dfrac{t}{2} - f$

Table 15 Profits table at $t>0$, $h>0$, with a_c near *and* a_f near c_f (FTTH-service-based competition)
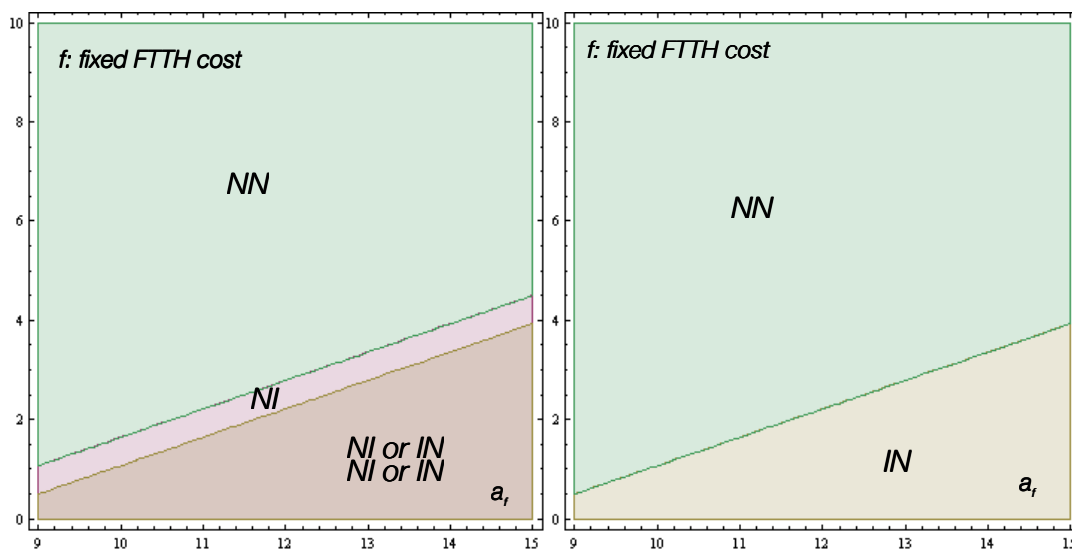

Figure 10 Investment incentives for *NN, NI, IN* and *II* as functions of a_c and *f* for t>0 (FTTH-service-based competition)

The lefthand region plot above indicates the following as a function of FTTH access charge and fixed FTTH cost:

- The "*II*" region is absent. When a_f~c_f, the FTTH access charge is close to cost, the rival prefers to buy access instead of investing in its own network.
- The *"IN"* region increases with the FTTH access charge a_f.

- The "*NI*" region a also increases with the FTTH access charge a_r. A higher copper access charge ($a_c > c$) encourages the rival to invest more in FTTH than the incumbent.
- The "*NN*" region occurs for higher FTTH deployment fixed costs *f*: when *f* is too high, neither firm invests in FTTH.

If only the incumbent has the financial capacity to invest in FTTH, the righthand region plot above shows that the incumbent's investment incentives increase with FTTH access charge.

It can be concluded that, in FTTH-service-based competition, FTTH investment incentives are not as significant as in FTTH-infrastructure-based competition. First, situation *II*, in which both firms invest, does not exist in a perfect competition (t=0) or a generic situation (t>0) where access charges are set at cost ($a_c \sim c$ and $a_r \sim c_r$). In situations *"IN"* and "*NI*", where only one firm invests, investment incentives increase with FTTH access charge.

5 Comparison of FTTH-infrastructure-based and FTTH-service-based competition in terms of nationwide FTTH coverage and social welfare

In this section, we will discuss the evolution of social welfare when consumers migrate from copper access network to FTTH network. To do so, the consumer surplus should be calculated for each of the situations "*NN*", "*NI*", *"IN* and *"II"*.

For nationwide FTTH coverage, we assume that the population is distributed as a concave function with respect to fixed FTTH deployment costs, $x = \dfrac{1}{5} \log\,(15f)$, as shown below.
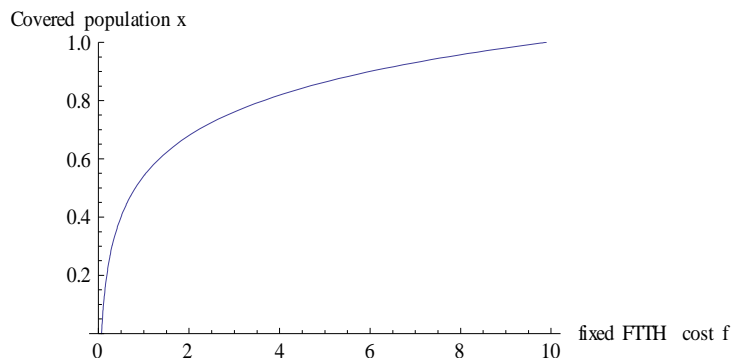

Figure 11 Populations covered as a function of fixed FTTH costs

Where x is percentage of population covered by FTTH with fixed cost *f.* With this heterogeneous population distribution, part of population is in situation *II*, part is in *NI* or *IN*, and part is in *NN*. We must therefore calculate the nationwide consumer surplus and social welfare, taking into account the population distribution for each situation (*NN, NI, IN, II*).

Without loss of generality, we will take the case of perfect competition with t=0 in FTTH-infrastructure-based and FTTH-service-based competition. The population is distributed

between **NI** and **NN**. x_1 is the percentage of the population covered by the rival (**NI**) and $(1 - x_1)$ is the percentage of population not covered by FTTH (**NN**).

$$x_1 = \frac{1}{5}\log(15 f_r^1)$$

$$CS_t = x_1 CS_{NI} + (1 - x_1)CS_{NN}$$

$$W_t = CS_t + x_1(\pi_i^{NI} + \pi_r^{NI}) + (1 - x_1)(\pi_i^{NN} + \pi_r^{NN})$$

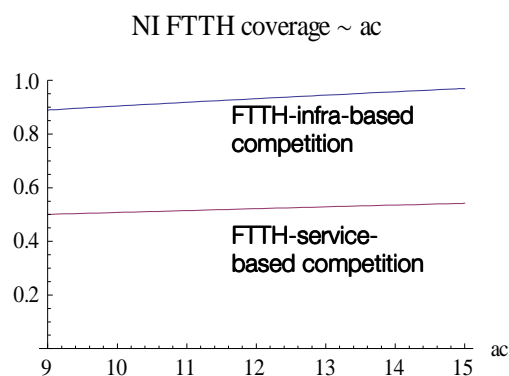Where **CSt** and **Wt** are respectively total nationwide consumers' surplus and social welfare.

NI FTTH coverage ~ ac

Figure 12 *NI* FTTH coverage as a function of a_c ($a_r=c_f=9$)

The curve above indicates that the FTTH coverage by the rival in situation "**NI**" in FTTH-infrastructure-based competition (blue curve) is higher than in FTTH-service-based competition (violet curve). FTTH coverage increases with copper access charge.
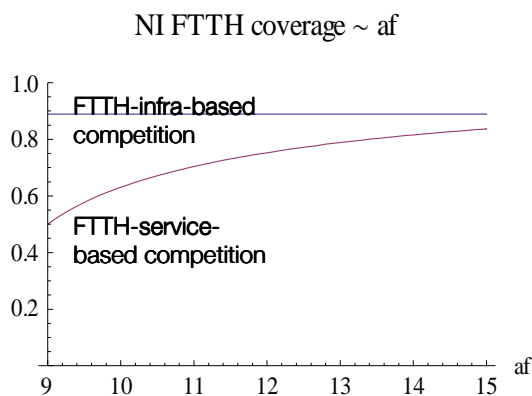
NI FTTH coverage ~ af

Figure 13 *NI* FTTH coverage as a function of a_f ($a_c=c=9$)

The curve above indicates that the FTTH coverage increases with FTTH access charge in FTTH-service-based competition.

- The curves on the right show that total social welfare in FTTH-service-based competition is maximized at a level of copper access charge above cost.
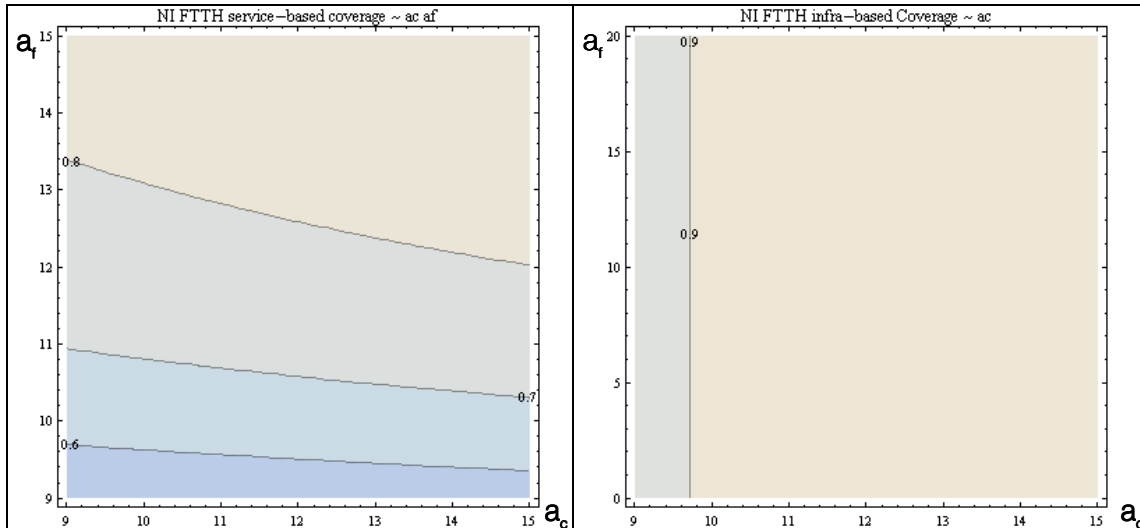
Figure 14 *NI* FTTH coverage in FTTH-service-based and FTTH-infrastructure-based competition
(see Appendix A: for *IN* FTTH coverage in FTTH-service-based and FTTH-infrastructure-based competition)

As a function of copper access charge in the x-axis and a function of FTTH access charge in the y-axis, the contour plot above shows the following.

- On the left, the contour plot shows that FTTH coverage in FTTH-service-based competition is very sensitive to the FTTH access charge a_f and increases with a_f. Coverage also increases with the copper access charge a_c but is much less sensitive to a_f.
- On the right, the contour plot shows that FTTH coverage in FTTH-infrastructure-based competition is only sensitive to the copper access charge a_c and increases with a_c.
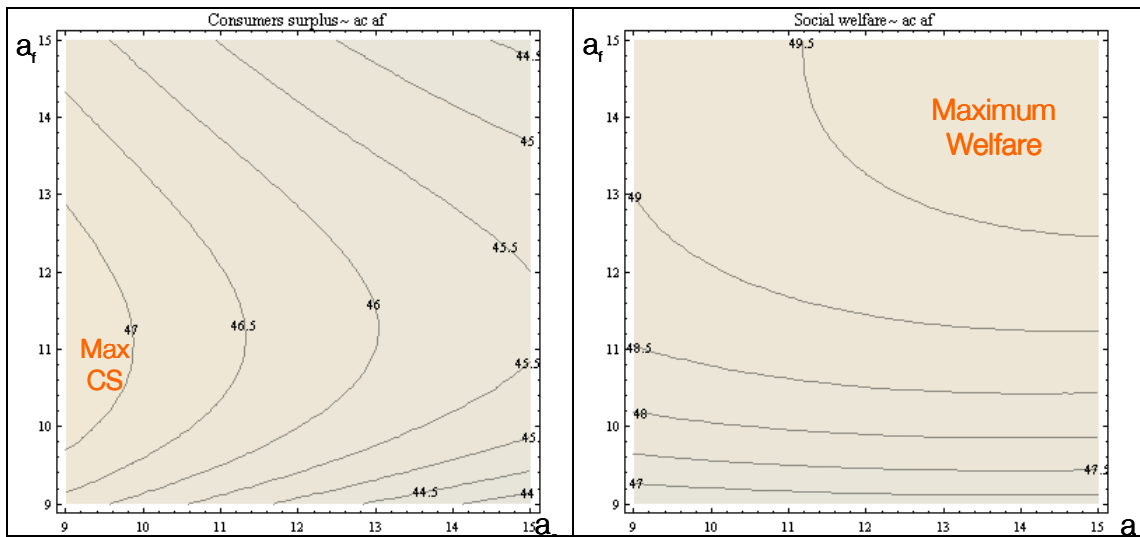

Figure 15 total consumer surplus and social welfare as functions of a_c and a_f in FTTH-service-based competition for *NI* FTTH coverage
(see Appendix B: total consumer surplus and social welfare as functions of a_c and a_f in FTTH-service-based competition for *IN* FTTH coverage)

As a function of copper access charge in x-axis and a function of FTTH access charge in y-axis, the contour plot above shows the following:
- On the left, the contour plot shows that the total consumer surplus in FTTH-service-based competition decreases with copper access charge and is maximized at a level of FTTH access charge which is higher than the marginal cost (c_f=9).
- On the right, the contour plot shows that total social welfare in FTTH-service-based competition is maximized at a level of both copper and FTTH access charge which is higher than their marginal costs (c=9, c_f=9).

6 Conclusion and further research

This study proposes a duopoly model (an incumbent and a rival) based on two–dimensional Hotelling method. By using "vertical product differentiation", we analyzed both intra- and inter-platform competition (Copper-Copper competition, FTTH-FTTH competition and copper-FTTH competition). Using the description of the utility function of copper and fiber broadband access, Nash equilibrium can be derived in a game where both firms compete on the prices of copper and fiber access after FTTH investment. The paper's originality consists in integrating intra-platform and inter-platform competition into a single model.

This model shows that when consumers are segmented copper access charge has a significant impact on broadband consumers' migration from copper to FTTH access. Lower access charge leads to a lower copper price equilibrium, meaning that consumers are encouraged to remain on copper access. To a certain extent, higher access charge leads to higher equilibrium prices for copper access, which encourages consumers to migrate toward FTTH access.

In FTTH-infrastructure-based competition, where fixed FTTH infrastructure costs are low, both the incumbent and the rival invest. Where fixed costs are higher, the rival or the incumbent invests alone. Finally, where fixed costs are very high, neither firm invests. In areas where only one firm invests, the incumbent may invest alone in FTTH whatever the level of the copper access charge, provided that its FTTH network is not open to competitors, meaning that only the incumbent can propose an FTTH offer. The rival may invest alone in FTTH only with a high copper access charge. In other words, the "*NI*" area where only the rival invests, increases with copper access charge. Maximum social welfare and FTTH coverage are achieved with copper access charge which are higher than cost. Their value depends on the difference in consumers' willingness to pay and marginal costs between copper and FTTH infrastructures.

In FTTH-service-based competition, the investment incentive for both firms (area "*II*") is absent if both access charges are regulated at marginal cost level. The "*NI*" and "*IN*" areas, where only one firm invests, increase with the FTTH access charge. However, investment incentives are much less sensitive to copper access charge, which do not play an essential role in investment incentives, unlike in FTTH-infrastructure-based competition. In order to maximize FTTH coverage, the consumer surplus and social welfare within FTTH-service-based competition, the optimal level for FTTH access charge should be set above marginal cost. The obligation to open its FTTH network to competitors reduces investment incentives even in areas where only one firm invests. The model determines the link between copper and FTTH access charge in order to maximize nationwide FTTH coverage, consumer surplus, and social welfare. In the long run, coverage seems to be the most important parameter because it is likely to

generate more technological progress, which dramatically increases social welfare over time (Jeanjean, 2010).

Further research will include the impact of an alternative and independent platform such as cable in the competition and pricing game, and the impact of copper pricing on alternative platforms.

Appendix A: *IN* FTTH coverage in FTTH-service-based and FTTH-infrastructure-based competition
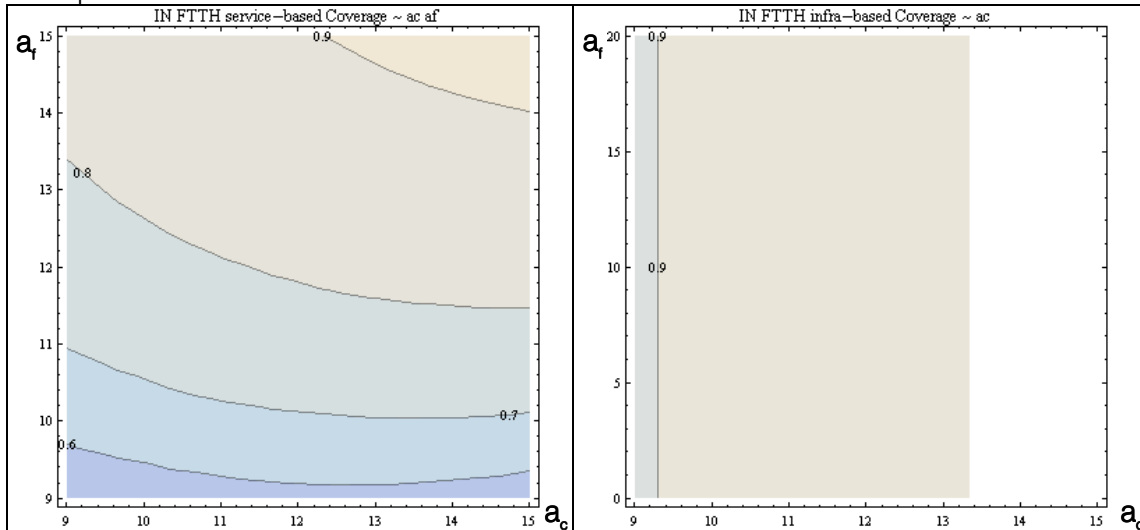
Appendix B: total consumer surplus and social welfare as functions of a_c and a_f in FTTH-service-based competition for incumbnt's FTTH coverage
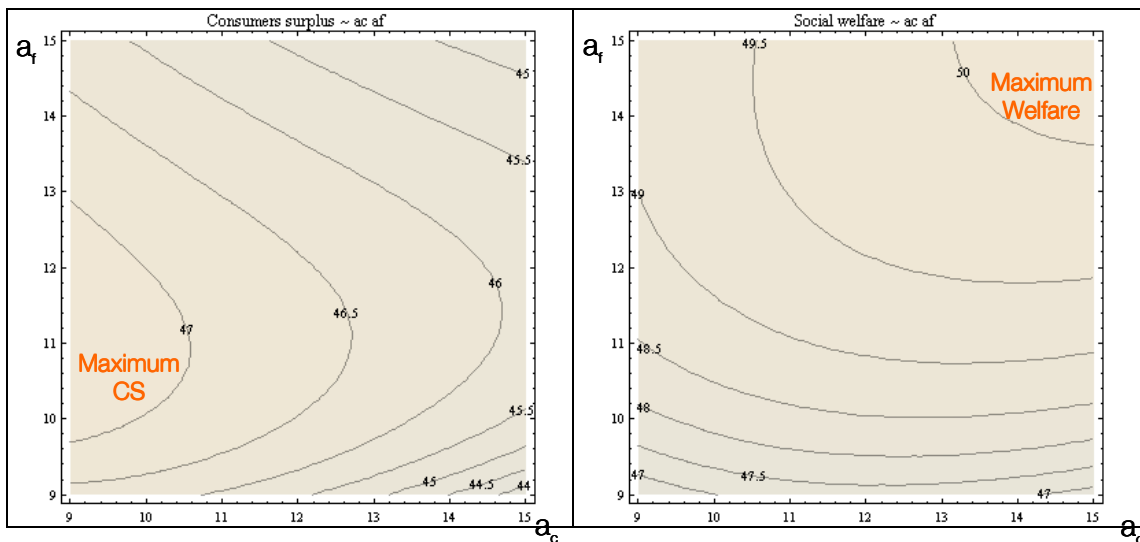
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