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João Macieira
Virginia Tech

Pedro Pereira
AdC and CEFAGE-UE

João Vareda
AdC and CEFAGE-UE

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Bundling Incentives in Markets with Product Complementarities: The Case of Triple-Play*

João Macieira† Virginia Tech

Pedro Pereira‡ AdC and CEFAGE-UE

João Vareda§ AdC and CEFAGE-UE

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Abstract

We analyze firms' incentives to bundle and tie in the telecommunications industry. As a first step, we develop a discrete-choice demand model where firms sell products that may combine several services in bundles, and consumers choose assortments of different types of products available from various vendors. Our approach extends standard discrete-choice demand models of differentiated product to allow for both flexible substitution patterns and to map demand for each choice alternative onto the demand for each service or bundle that a firm may sell. We exploit these properties to examine bundling behavior when firms choose: **(i)** prices, and **(ii)** which products to sell. Using consumer-level data and survey data from the Portuguese telecommunications industry, we estimate our demand model and identify firm incentives to bundle and tie in this industry. We use the model to perform several policy related conterfactuals and evaluate their impact on prices and product provision.

Key Words: *Bundles, Discrete-Choice Model, Equilibrium Simulation, Differentiated Product, Consumer Level Data.*

JEL Classification: D43, K21, L44, L96.

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†Department of Economics, Virginia Tech. 3044 Pamplin Hall (0316), Blacksburg, VA 24060. E-mail: joaom07@vt.edu.

‡AdC, Avenida de Berna, n^o 19, 7^o, 1050-037 Lisboa, Portugal. E-mail: pedro.br.pereira@gmail.com.

§AdC, Avenida de Berna, n^o 19, 7^o, 1050-037 Lisboa, Portugal. E-mail: joao.vareda@concorrencia.pt.

1 Introduction

Bundling is becoming an important characteristic of the telecommunications industry. An increasing number of households prefers to consume telecommunication services in bundles, rather than separately. In addition, telecommunications firms increasingly base their marketing strategies on these products. The increasing importance of bundles might be the result of many different causes, such as: technological progress, changes in consumption habits, or shifts in the strategic environment. For a discussion of these potential motives see Pereira and Vareda (2011, 2013).

Other examples of industries where bundling is prevalent include: insurance, software, newspapers, music or air transportation. While the theoretical literature on bundling, reviewed in Section 2, has been extensively developed over the last decades, empirical work on bundles has only recently experienced important advances, e.g., Chu et al. (2011), Crawford and Yurukoglu (2012), Gentzkow (2007), Pereira et al. (2013). An important void in this literature is the lack of modeling of both: **(i)** the demand for product assortments by consumers, and **(ii)** the joint decisions of pricing and supply of products by firms. In the article we fill in this void.

We propose a differentiated product equilibrium model that has three important characteristics, following Pereira et al. (2013). First, there is a set of basic ***services***. Second, firms sell ***products*** that may consist of these individual services, or may combine several of them in bundles. Third, consumers choose ***alternatives***, i.e., choose assortments of products that may consist of a single product sold by a given firm, or may combine several products, possibly sold by different firms.

For the demand side, we propose a discrete-choice model that builds upon and extends the differentiated product frameworks of Berry et al. (1995) and Nevo (2001), where consumers choose alternatives as in Pereira et al. (2013).[1] More specifically, we replace the Mixed Logit framework of Berry et al. (1995) and Nevo (2001) by an extension of the Cross-Nested Logit model with random effects.[2] In particular, we use the Cross-Nested Logit model with the parametrization of Bresnahan et al. (1997). For a general discussion of the properties of the Cross-Nested Logit model see, e.g., Bierlaire (2006).[3] The resulting framework allows for flexible substitution patterns, allows handling a potentially large number of consumer choice alternatives, and allows mapping the demand for each choice alternative onto the demand for each service or bundle that a firm may sell. By letting each service be a nest,

[1] More generally, our methodological approach draws on the discrete choice literature, represented by, e.g., Domencich and McFadden (1975) and McFadden (1974, 1978, 1981).

[2] See Grigolon and Verboven (2013).

[3] The Cross-Nested model belongs to the Generalized Extreme Value class introduced by McFadden (1978). See also Fosgerau, McFadden, and Bierlaire (2010), Wen and Koppelman (2001), and Koppelman and Sethi (2007). Previous applications of the Cross-Nested Logit model in economics include Adams et al. (2007), Bresnahan et al. (1997), Pereira et al. (2013) and Small (1987).

our framework includes Berry et al. (1995) and Nevo (2001) as a particular case, and allows to control for having alternatives embodying different services. In addition, allowing for consumer heterogeneity in tastes enables the examination of the role of this feature on the incentives to bundle, namely as a means to replicate price discrimination.

For the supply side, we allow firms to choose both: **(i)** prices, and **(ii)** which products to supply, individual services and bundles. Firms acknowledge that consumers acquire their products as part of a product assortment, by letting the share of each product equal the sum of purchase probabilities where that product is present. In addition, the decision to sell or not a single service or a bundle impacts not only firms' profits, but also the set of alternatives available to consumers. Our supply model allows firms to engage in various forms of pricing such as: component pricing, pure bundling, mixed bundling and tying.

We estimate the model using consumer-level data and survey data from the Portuguese telecommunications industry, focusing on triple-play bundles, i.e., products that include: **(i)** fixed telephony, **(ii)** fixed broadband access to the internet and **(iii)** subscription television. Since triple-play products embody three services and that one observes these services being supplied in different combinations across firms and markets, this is a unique opportunity to study firms' incentives to bundle. More specifically, we exploit the fact that in the context of triple-play products, sometimes firms sell services in bundles that are unavailable individually, to identify firm incentives to bundle products.

Bundling may be motivated by potential cost savings from selling several products jointly. We use model estimates to identify product marginal costs and test for bundling cost synergies.[4] Our results do not reject the hypothesis of no bundling cost synergies in this industry. We examine alternative incentives to bundle and tie identified in the literature by using the estimated model to perform counterfactuals. While our model allows simulating the welfare impact of many policy issues of practical interest, e.g., mergers or tying bans, we focus on two counterfactuals.

First, we examine bundling and pricing behavior of firms in response to variations in consumer heterogeneity of the price coefficient of demand. An explanation advanced by the theoretical literature is that bundling is used to replicate price discrimination by exploiting consumer heterogeneity in price sensitivity. We assess the plausibility of this claim by analyzing how the simulated market equilibrium changes when the variance of the price coefficient of demand increases. Our simulation results indicate that firms do respond to higher product sensitivity through introduction of more products in the market, including but not limited to bundles. In addition to price reduction, we observe some quality reshuffling from single-play products to double- and tripleplay bundles. This quality reshuffling takes the form of single-play products with lower attribute values (e.g. lower number of channels, lower bandwidth size) while bundle products display higher attribute values.

[4]Production synergies are different from selling synergies. Bundling cost synergies refers to the latter.

Second, we examine bundling and pricing behavior by firms in response to different market structures. The theoretical literature has argued that, depending on the circumstances, bundling can be used either to decrease or increase the level of competition. We assess the plausibility of this claim by analyzing how the simulated market equilibrium changes when the number of firms in the industry increases.We examine how a firm chooses prices and products to supply in a representative market when **(i)** it is a monopolist in that market, and **(ii)** it competes in duopoly with a similar firm. Our simulation results indicate that moving from monopoly to duopoly leads to both lower prices and introdution of products not sold under monopoly. In contrast to the higher consumer heterogeneity scenario, increased competition leads to higher average quality of all products in the form of higher values for product attributes valued by consumers. Our two counterfactuals indicate that firms do consider product commercialization decisions when pricing bundles, yet most of the thoery literature on bundles does not allow for firm decisions on product availability. Our results illustrate its importance in practice.

The rest of the article is organized as follows. In Section 2 we revise the literature related to our paper. Section 3 gives an overview of the Portuguese triple-play industry. Section 4 outlines our structural model of demand and supply. Section 5 discusses our data set and the estimation of the model. We present and discuss estimation results in Section 6. Section 7 addresses the illustration of model properties via computer simulation. Section 8 concludes.

2 Literature Review

Our article relates to three large bodies of literature. The first consists of the literature that develops estimable equilibrium models for differentiated product industries, pioneered by Bresnahan (1987) and extended by Berry (1994), Berry et al. (1995) and Nevo (2000, 2001).[5] The second consists of the literature that models and estimates the choice over bundles, which includes Augereau et al. (2006) for internet standards, Gandal et al. (2013) and Riordan (2004) for personal computer office software, Gentzkow (2007) for print and online newspapers, and Pereira et al. (2013) for telecommunications services.[6] The third consists on the literature on bundling and tying, which includes both theoretical and empirical work.

The theoretical literature on bundling and tying focuses on firms' incentives, which are typically related to preference or technology complementarities, price discrimination, and firm strategic behavior.[7] Stigler (1963) showed, through an example, that mixed bundling

[5] More generally, our methodological approach draws on the discrete choice literature, represented by, e.g., Domencich and McFadden (1975), McFadden (1974, 1978, 1981).

[6] Liu et al. (2010) using the model of Gentzkow (2007) find strong complementarities between subscription television and fixed broadband/cable modem and between fixed voice and fixed broadband/DSL.

[7] For a discussion of the Leverage Doctrine and the One-Profit Doctrine see, e.g., Bowman (1957), Director and Levi (1956) and Schmalensee (1982).

can be profitable for a monopolist, if consumer valuations are negatively correlated across goods. Under these circumstances, bundling can be used to reduce consumer heterogeneity and has an effect similar to price discrimination, thereby facilitating the extraction of consumer surplus. Afterwards, several authors, e.g., Adams and Yellen (1976), Chen and Riordan (2011), Fang and Norman (2006), Geng et al. (2005), McAfee et al. (1989), Salinger (1995), Schmalensee (1984), showed with various degrees of generality that this intuition holds as long as valuations are not perfectly positively correlated. Carbajo et al. (1990), Chen (1997) showed that pure bundling can be used as a product differentiation strategy that reduces the intensity of competition. However, depending on consumer preferences, bundling may actually increase the level of competition, as shown by Anderson and Leruth (1993) for the case of logit preferences. Hurkens et al. (2013) investigate in the context of an asymmetric duopoly whether bundling increases or reduces competition. Carlton and Waldman (2011, 2002), Carlton et al. (2010), Nalebuff (2004), Peitz, M. (2008), Whinston (1990) showed that bundling can be used to foreclose the market, or more generally to create, preserve or extend monopoly positions. Chu et al. (2011) argue that mixed bundling can be very complex even with a small number of individual products and evaluates to what extent simpler pricing schemes, such as bundle-size pricing, allow a monopolist to capture a substantial part of the profits of mixed bundling.

We provide an estimable differentiated product equilibrium model, where consumers choose among a set of alternatives, that may or may not consist of bundles, and where firms' pricing and supply decisions, that may or may not include bundles, are the result of optimizing behavior. The model enables evaluating numerically the empirical relevance of the behavior predicted by the theoretical literature.

There is a an emerging empirical literature on the welfare impact of bundling in several industries, which helps to put in perspective the wide range of welfare implications of the various theoretical justifications for bundling. Byzalov (2010) analyzed the welfare impact of various restrictions to bundling of channels for the cable television industry, using consumer level data and taking upstream prices as exogenous. Chu et al. (2011) analyzed the profitability of simple pricing alternatives to mixed bundling for the theatre industry. Crawford (2008) analyzed the discriminatory incentives for bundling in the cable television industry. Crawford and Yurukoglu (2012) estimated the welfare effects of unbundling in the retail cable television industry, using firm level data and endogeneizing upstream prices. Derdenger and Kumar (2012) analyzed hand-held video games. Haas-Wilson (1987) analyzed the impact of state legislation on tying for the contact lens industry. Ho et al. (2012) analyzed full-line forcing in the video rental industry. Pereira et al. (2013) analyzed if bundles of subscription television, fixed broadband and fixed voice are a relevant product market in the sense of competition policy, using consumer level data. Shiller and Waldfogel (2011) analyzed the welfare impact of various forms of pricing for the music industry.

The growing importance of triple-play products poses several problems for competition authorities and sectoral regulators, discussed in Pereira and Vareda (2011, 2013) and Pereira et al. (2013), which need to be evaluated empirically. However, direct usage of conventional differentiated product models, e.g., Berry et al. (1995) and Nevo (2001), to examine structural market changes, such as mergers and acquisitions, is problematic. In this article we take a step to fill in that void by developing a estimable differentiated product equilibrium model that accommodates bundling behavior.

3 The Portuguese Industry

In Portugal, in the early 1990s, both domestic television and telecommunications were state-owned monopolies. Later on that decade, several free-to-air television channels and cable television firms were licensed and the telecommunications incumbent, Portugal Telecom (PT), was privatized.[8] The telecommunications industry was further liberalized in 2000.

Initially, entrants based their offers of fixed voice and broadband access services in the wholesale access to PT's cooper wire network. Later, as they obtained a substantial customer base, entrants resorted to the unbundled access to PT's local loop. After 2006 there was a large increase in the number of unbundled loops. As a consequence, many innovative products, for instance bundles, were introduced in the market. In the meanwhile, some entrants invested in their own infrastructures, increasing further their autonomy. In November 2007, ZON, a cable television firm, was spun-off from PT. This was an important change in the Portuguese industry. ZON, using its cable television network, started to compete with PT, using its telephone network.[9] Recently, PT initiated the deployment of a fiber-optic network, while ZON upgraded its cable network by installing DOCSIS 3.0.

The other relevant firms in the industry include AR Telecom, Cabovisão, Optimus and Vodafone. AR Telecom began operations in 2005, basing its products mainly on fixed wireless access technology. Cabovisão, a cable television firm, was created in 1995. Optimus, originally a mobile telecommunications firm, entered the fixed services business in 2000 using local loop unbundling, with access via Digital Subscriber Line (DSL). After 2008 it also started deploying its fiber-optic network. Vodafone, originally a mobile telecommunications firm, entered the fixed services business in 2000, using local loop unbundling, with access by DSL.

In November 2011, AR Telecom exited the market and passed its customers to ZON. In January 2013, ZON proposed a merger with Optimus. The operation is under evaluation by the Portuguese Competition Authority.

Our data set, described below, contains information only for 2009. In that year, the

[8] Private free-to-air channels were licensed in 1992, PT was privatized in 1996, cable television licenses were issued in 1997 and in 1999 cable television firms were authorized to offer telecommunications services.

[9] For more details see Pereira and Vareda (2011, 2013).

penetration rate per inhabitants of fixed telephony was 40%. After a long period of decline, the penetration rate of fixed telephony started to increase again, slightly. Also in 2009, the penetration rate per households of subscription television was 45%. Of these subscribers, 57.4% used cable and 23.2% Direct to Home (DTH) technology.[10] Finally, in 2009, the penetration rate per inhabitants of fixed broadband was 18%. Of these subscribers, 57% used DSL and 40% used cable modem.

Table 1 presents the markets shares of the largest telecommunications firms in 2008 and 2009 for each type of service.

[*Table* 1]

Telecommunications bundles were first offered in Portugal in 2004 through cable television networks. Afterwards, several firms launched similar products using fixed telephone networks, either through local loop unbundling or their own networks.

4 Model

4.1 General Considerations

In this Section, we propose an estimable differentiated product equilibrium model in the spirit of Berry et al. (1995) and Nevo (2000, 2001), but with four differences. First, there is a set of basic ***services*** and firms sell ***products*** that may combine several of these services. In our application there are three services: **(i)** fixed telephony (FV), **(ii)** fixed broadband access to the internet (BB) and **(iii)** subscription television (TV). Firms may sell three types of products: single-play products, which include only one service, double-play products, which include two services, and triple-play products, which include the three services. Second, consumers choose among ***alternatives*** that may combine several products, possibly sold by different firms. For example, a consumer may choose a double-play product TV+BB supplied by a given firm, while also purchasing a single-play product FV supplied by another firm. Third, we model consumer preferences through a Cross-Nested Logit (CNL) model with random effects. The fact that the set of all consumer alternatives consists of the set of all available product assortments, our product demand model should allow for correlation in alternative preference shocks since many alternatives contain similar products. To this end, we consider Bresnahan et al.'s (1987) parametrization of the CNL model. In contrast to conventional models of market segmentation, such as the Nested Logit, the PD-GEV allows for overlapping product nests. This property allows us to control for closer substitution of alternatives lying within a segment, while allowing for choices containing products with various services to be present in several segments. Fourth, aside from prices, firms choose

[10] In Portugal there are no independent satellite television firms. Two of the telecommunications or television firms offer satellite television services as complements to their other services in the regions not covered by their physical networks.

which products to sell in each market. This allows for strategic supply behavior in markets with bundles. For example, firms may decide not to sell a certain product consisting only of single service to encourage consumers to choose another product, which bundles that service with other services.

We denote markets by subscript $t = 1, ..., T$, services by subscript $i = 1, ...I$, products combining services by subscript $j = 1, ..., J$, alternatives combining products by subscript $a = 0, 1, ..., A$, where $a = 0$ is the alternative of buying no product, and firms by subscript $f = 1, ...F$.

We follow the convention of denoting by $\mathbf{x} := (x_1, ..., x_n)$, a n-dimensional vector of real numbers, x_j, and letting $\mathbf{x}_{-j} := (x_1, ..., x_{j-1}, x_{j+1}, ..., x_n)$.

4.2 Supply

We assume that firms simulateously decide both: **(i)** which products to supply, and **(ii)** product prices. Denote by $d_{jt} \in \{0, 1\}$ the decision of the firm that owns product j to sell it in market t and by $p_{jt} \in \mathbb{R}^+$ the product's price. Denote by c_{jt} the marginal cost of product j in market t, by C_{ft} the fixed cost of firm f in market t, by M_t the number of consumers in market t, and by $s_{jt}(\mathbf{p}_t, \mathbf{d}_t)$ the market share of product j in market t. Firm f's profit is:

$$\Pi_{ft}(\mathbf{p}_t, \mathbf{d}_t) = \sum\nolimits_{j \in \mathcal{F}_f} d_{jt}(p_{jt} - c_{jt}) M_t s_{jt}(\mathbf{p}_t, \mathbf{d}_t) - C_{ft}. \tag{1}$$

In each market t, each firm f maximizes profits by choosing: **(i)** which products to sell, and **(ii)** their prices. The the first-order condition for price p_{jt} is:

$$d_{jt} s_{jt}(\mathbf{p}_t, \mathbf{d}_t) + \sum\nolimits_{j \in \mathcal{F}_f} d_{kt}(p_{jt} - c_{jt}) \frac{s_{kt}(\mathbf{p}_t, \mathbf{d}_t)}{\partial p_{jt}} = 0. \tag{2}$$

Firm f will sell product j in market t, if and only if, the decision to sell it generates a profit higher than the decision of not to sell it, i.e., if and only if, $\forall j \in \mathcal{F}_f$, $f = 1, ..., F$:

$$\Pi_{ft}(\mathbf{p}_t, \mathbf{d}_t) = \max \left\{ \Pi_{ft}(\mathbf{p}_t, 0, \mathbf{d}_{-jt}), \Pi_{ft}(\mathbf{p}_t, 1, \mathbf{d}_{-jt}) \right\}. \tag{3}$$

For all $j, k = 1, ..., J$, define the $J \times J$ ownership matrix $\mathbf{H}$, whose generic element is:

$$H_{jk} = \begin{cases} 1 & \text{if} \quad \exists f : \{j, k\} \subset \mathcal{F}_f \\ 0 & \qquad\qquad \text{otherwise.} \end{cases} \tag{4}$$

In addition, denote the $J \times J$ matrix containing derivatives of market shares with respect to prices by $\mathbf{\Delta}$, whose generic element is $\Delta_{jk} = -\frac{\partial s_k}{\partial p_{jt}}$. Denote by $*$ the Hadamard product, i.e., the element-by-element matrix product. The system defined in (2) can be written in matrix form as:[11]

$$\mathbf{d}_t * \mathbf{s}_t(\mathbf{p}_t, \mathbf{d}_t) - (\mathbf{H} * \mathbf{\Delta}) \left[\mathbf{d}_t * (\mathbf{p}_t - \mathbf{c}_t) \right] = 0. \tag{5}$$

[11] This system is similar to those of Bresnahan (1987), Berry et al. (1995), Nevo (2000, 2001), except that we control for the decision of whether to supply a product.

We follow the literature in assuming the existence of a pure-strategy, Nash equilibrium, which in our case is both in prices and commercialization decisions. That is, a Nash Equilibrium in each market $t = 1, ..., T$ consists of a vector of product supply decisions $d_t = (d_{1t}, ..., d_{Jt})$ and non-negative prices $p_t = (p_{1t}, ..., p_{Jt})$ that solve both (2) and (3) for all $j = 1, ..., J$.

4.3 Demand

The complete definition of system (5) requires additional structure to relate product market shares and consumer demand for product assortments. We assume that consumers choose among alternatives, i.e., assortment of products sold by firms aimed at providing several different services. For example, an assortment of subscription TV with 30 channels and broadband internet with bandwidth of 30 Mbps provides two services: subscription TV and broadband internet. Denote by u_{hjt} the utility derived by consumer h from product j included in alternative a, by ϵ_{hat} a preference shock of consumer h for alternative a, by $\mathbf{X}_{jt}$ a $K \times 1$ vector of observed characteristics of product j, by ξ_j is a market-wise mean consumer valuation of product characteristics unobserved to the researcher, by $\Delta\xi_{jt}$ the market-specific deviation from ξ_j, and by $(\alpha_h, \boldsymbol{\beta}_h)$ a vector of consumer-specific taste parameters for observed attributes $(p_{jt}, \mathbf{X}_j)$.

The utility that consumer h derives from alternative a is:

$$u_{hat} = \sum\nolimits_{j \in a} u_{hjt} + \epsilon_{hat}. \tag{6}$$

In addition, u_{hjt} is given by:

$$u_{hjt} = \mathbf{X}_j \boldsymbol{\beta}_h - \alpha_h p_{jt} + \xi_j + \Delta\xi_{jt}; \tag{7}$$

We assume that the joint distribution of $(\alpha_h, \boldsymbol{\beta}_h)$ is:

$$\begin{pmatrix} \alpha_h \\ \boldsymbol{\beta}_h \end{pmatrix} = \begin{pmatrix} \alpha \\ \boldsymbol{\beta} \end{pmatrix} + \mathbf{v}_h; \tag{8}$$

where $\mathbf{v}_h \sim N(\mathbf{0},\boldsymbol{\Psi})$ and $\boldsymbol{\Psi}$ is a $(K+1) \times (K+1)$ strictly positive, diagonal matrix.

Let δ_{at} be the mean utility from choosing alternative a and μ_{hat} be the portion of utility that depends on consumer-specific components:

$$\delta_{at} = \sum_{j \in a} X_j \boldsymbol{\beta} - \alpha \sum_{j \in a} p_{jt} + \sum_{j \in a} \xi_j + \sum_{j \in a} \Delta\xi_{jt}, \tag{9}$$

$$\mu_{hat} = \left[\sum_{j \in a} p_{jt}, \sum_{j \in a} X_j \right]' * \mathbf{v}_h. \tag{10}$$

In the spirit of Berry et al. (1995) and Nevo (2000,2001), we rewrite u_{hat} as:

$$u_{hat} = \delta_{at} + \mu_{hat} + \epsilon_{hat}; \tag{11}$$

As the utility derived from alternative $a = 0$ is not identified, we let $\delta_{0t} = 0$ and $\mu_{h0t} = 0$. That is, the utility from this outside alternative has zero mean and is given by $u_{h0t} = \epsilon_{h0t}$, where ϵ_{h0t} is the outside alternative shock.

Given the set of alternatives available in market t, each consumer chooses the alternative that maximizes his utility. We assume further that, for each consumer the vector of alternative preference shocks $(\epsilon_{h0t}, \epsilon_{h1t}, ..., \epsilon_{hAt})$ follows a Generalized Extreme Value (GEV) distribution. That is, as in McFadden (1978), the joint distribution of the preference shocks vector is:

$$F(\epsilon_{h0t}, ..., \epsilon_{hAt}) = \exp\left(-G(e^{-\epsilon_{h0t}}, ..., e^{-\epsilon_{hAt}})\right); \tag{12}$$

where $G(\cdot)$ is a nonnegative, homogenous of degree one function mapping set $\mathbb{R}^{A_t+1}$ onto $\mathbb{R}_0^+$, whose partial derivative with respect to the term $e^{\delta_{at}+\mu_{hat}}$ is denoted by $G_a(\cdot)$.[12] Under this assumption, the probability that consumer h chooses alternative a in market t is:

$$s_{hat} = \frac{e^{\delta_{at}+\mu_{hat}} G_a(e^{\delta_{0t}+\mu_{h0t}}, ..., e^{\delta_{At}+\mu_{hAt}})}{G(e^{\delta_{0t}+\mu_{h0t}}, ..., e^{\delta_{At}+\mu_{hAt}})}. \tag{13}$$

The fact that consumers choose among combinations of available products, possibly offered by different suppliers, motivates letting our product demand model allow for correlation in alternative preference shocks. To this end, we build upon and extend the model proposed by Bresnahan et al. (1997), where the GEV model is restricted to allow for market segmentation along Principles of Differentiation (PDs), or nests. That is, we consider Bresnahan et al.'s (1997) parametrization of the CNL model. In contrast to conventional models of market segmentation (e.g.Nested Logit), the PD-GEV allows us to control for closer substitution of alternatives lying within a segment, while allowing for choices containing products with various services to be present in several segments.

We consider nests for three services: **(i)** FV, **(ii)** TV and **(iii)** BB. Each nest implies two segments: one with only alternatives containing a product where that service is provided, and another segment with only alternatives where that service is absent. For example, the segment FV contains alternatives that include fixed-voice telephony service, and the segment NFV contains only alternatives without fixed-voice service, except the outside alternative. The segments pairs (TV, NTV) and (BB, NBB) are defined analogously for subcription television and fixed broadband, respectively. We denote the set of segments by $D = \{FV, NFV, TV, NTV, BB, NBB\}$ and we restrict each pair of segments to have the same segment parameter, whose value lies between 0 and 1. That is, the segments FV and NFV have the same segment parameter ρ_{FV}, while TV and NTV have segment parameter ρ_{TV} and both BB and NBB share the segment parameter ρ_{BB}.[13] Under these conditions, we compose the function $G(\cdot)$ as:

$$G(e^{\delta_{0t}+\mu_{h0t}}, ..., e^{\delta_{At}+\mu_{hAt}}) = \sum\nolimits_{d\in D} \gamma_d \left(\sum\nolimits_{a\in d} e^{(\delta_{at}+\mu_{hat})/\rho_d}\right)^{\rho_d} + e^{\delta_{0t}+\mu_{h0t}}; \tag{14}$$

[12] We forward the reader to McFadden (1978) for full set of assumptions function $G(\cdot)$ must satisfy.

[13] For a discussion, see Bresnahan et al. (1997) and Wen and Koppelman (2001).

where γ_d are scaling parameters defined by:

$$\gamma_d = \frac{1-\rho_d}{3-\rho_{FV}-\rho_{TV}-\rho_{BB}}. \tag{15}$$

Denote by $P_{ah|d}$ the probability that consumer h chooses alternative a given that he is choosing from segment d, and by P_{dh} the probability that consumer h chooses an alternative from segment d. Under this specification, the consumer choice probability in (13) simplifies to:

$$s_{hat} = \sum\nolimits_d P_{ah|d}P_{dh} = \sum\nolimits_d \left(\frac{e^{(\delta_{at}+\mu_{hat}/\rho_d)}}{\sum_{k\in D} e^{(\delta_{kt}+\mu_{hkt})/\rho_d}}\right)\left(\frac{\gamma_d\left(\sum_{k\in D} e^{(\delta_{kt}+\mu_{hkt})/\rho_d}\right)^{\rho_d}}{G(e^{\delta_{0t}+\mu_{h0t}},...,e^{\delta_{At}+\mu_{hAt}})}\right). \tag{16}$$

Consumer heterogeneity is completely characterized by the random coefficient specification in (8). For ease of exposition, let $\boldsymbol{\theta}_h = (\alpha_h, \boldsymbol{\beta}_h)$, and denote $F(\boldsymbol{\theta}_h)$ as its CDF. Then the aggregate market share for alternative a in market t is:

$$s_{at} = \int s_{hat}(\boldsymbol{\theta}_h)dF(\boldsymbol{\theta}_h). \tag{17}$$

In is convenient to map markets shares of alternatives defined in (17) onto product market shares. For each product j, its market share is the sum of the shares of all alternatives where product j is included:

$$s_{jt} = \sum\nolimits_{a=1}^{A} \mathbf{1}\{j\in a\}s_{at}. \tag{18}$$

The total price of alternative a, denoted p_{at}, is the sum of the prices of the products that are part of alternative a:

$$p_{at} = \sum\nolimits_{j=1}^{J} \mathbf{1}\{j\in a\}p_{jt}. \tag{19}$$

For $j = 1,...,J$ and $a = 1,...,A$, define the $A\times J$ matrix $\boldsymbol{\Gamma}$, which controls for the presence of products in an alternative, and whose generic element is:

$$\Gamma_{aj} = \begin{cases} 1 & \text{if} \quad j\in a \\ 0 & \text{otherwise.}\end{cases} \tag{20}$$

Let $\mathbf{p}_t^{Alt}$ denote the vector of total price of alternatives, as defined in (19). Also, Denote by S_t^{Alt} the vector of market shares of alternatives defined in (17). Matrix $\boldsymbol{\Gamma}$allows us to write the product market share and alternative price vectors, respectively, as:

$$S_t = \boldsymbol{\Gamma}' S_t^{Alt}, \tag{21}$$

$$\mathbf{p}_t^{Alt} = \boldsymbol{\Gamma}\mathbf{p}_t. \tag{22}$$

The system of first-order conditions in prices defined in (5) involves a matrix $\boldsymbol{\Delta}$ of derivatives of product demand with respect to product prices. This matrix can also be written as a function the demand for alternatives. For any two alternatives a,$b = 1,...,A$ define the $A\times A$ matrix $\boldsymbol{\Delta}^{Alt}$, whose generic element is $\Delta_{ab}^{Alt} = -\frac{\partial s_b}{\partial p_{at}}$. It follows from the definitions above that $\boldsymbol{\Delta} = \boldsymbol{\Gamma}'\boldsymbol{\Delta}^{Alt}\boldsymbol{\Gamma}$.

5 Econometric Implementation

The goal of model estimation is to recover the firms' demand and costs parameters, as well as the random coefficient distributions. To this end, we combine the approaches of Berry et al. (1995) and Nevo (2000, 2001) by matching data information to model predictions, but with some differences. First, our supply system differs from the one in those articles, as it involves both decisions on product supply in each market and alternative – rather than product – market shares. Second, our demand system acknowledges that consumers purchase combinations of products. We start by describing the data used in the estimation and then turn to estimation details.

5.1 Data

5.1.1 Data Request

We obtained data of the last quarter of 2009 from six Portuguese electronic communication firms, which accounted in December 2009 for 99% of triple-play customers. For confidentiality reasons, we will refer to these firms as $f_1, ..., f_6$. The information obtained consisted of data about: **(i)** the contract, **(ii)** the product, **(iii)** the client and **(iv)** monthly expenditures. The characteristics of the contract are: the monthly fee, discounts or joining offers, the commencement date of the contract, and the characteristics of the product. The characteristics of the product are: the brand name, the number of normal and premium television channels and the possibility of access to video-on-demand, if the product included subscription television, bandwidth, traffic limits, number of e-mail accounts and the possibility of mobile broadband, if the product included fixed broadband access to the Internet, and the tariff plan for fixed telephony. The characteristics of the client are: age, length of the contract and residential postal code. We also obtained billing information for the last quarter of 2009, with full detail of invoices, including the fixed monthly fee and variable components, e.g., movie rentals, channel rentals, internet traffic above contracted limits, expenditure on telephone calls and minutes of conversation. Finally, we obtained the total number of clients for each product offered, and the geographical availability of each product. This data was complemented with information from the sectoral regulator, ICP-ANACOM, drawn from the survey "Inquérito ao consumo dos serviços de comunicações electrónicas - População residencial – Dezembro de 2009", from, hereon "Inquérito ao consumo", which characterizes the typical national consumer of electronic communication services.

5.1.2 Products, Markets and Choice Alternatives

We define a ***product*** as a combination of fixed-voice telephony, subscription channels and broadband internet services supplied by a single firm. A single-play product includes one *service*, a double-play product includes two services and a triple-play product includes the three services. Table 2 details the possible combinations of: services, forms of acquisition and firms.

$[Table\ 2]$

Our empirical model considers product shares and prices per market $t = 1, ..., T$. We define a market as a statistical NUTS3 region and consider a total of 30 NUTS3 regions.[14] The information from *Inquérito ao consumo* allowed us to relate the electronic communication services consumed by households to the way they are acquired, and to obtain the percentage of households that do not consume any of these services. Table 3 presents the distribution of services by type of bundle in 2009.[15]

$[Table\ 3]$

This information, and the data obtained from firms, allowed us to determine the distributions of the services per household and the market shares per firm for each service. We used this information along with the choice-based sample information on consumer purchases by firm in each region to derive regional market shares.

We compute the average product price in each market as the average monthly fee, net of all discounts, that costumers from a given NUTS3 region pay for the product. Market size and shares are computed using both the information on product sales available in each region and information on number of households in each NUTS3 region, as reported by *Instituto Nacional de Estatistica* (INE) - the Portuguese National Statistics Institute. The data used in the estimation consists of a total of $1,083$ product/market observations.[16] To distinguish single-, double- and triple-play products we form dummy variables to control for bundles. The observed characteristics of a product therefore consist of: **(i)** FV dummy variable, **(ii)** BB dummy variable, **(iii)** number of offered channels, if the product contains TV, **(iv)** bandwidth, in Mbps, in case the product includes BB, **(v)** dummy variables for double-play bundles $FV+TV$, $FV+BB$ and $TV+BB$, and **(vi)** triple-play bundle dummy variable. Table 4 presents average values of prices, marjket shares and characteristics by type of product:

$[Table\ 4]$

There is a total of 117 different products, yet product availability differs by region. The average market share of single-play products is bigger than double-and triple-play.

[14] The Nomenclature of Units for Territorial Statistics (NUTS) was created by the European Office for Statistics (Eurostat) as a single hierarchical classification of spatial units used for statistical production across the European Union. It is comparable to Metropolitan Statistical Area (MSA) classification in the USA.

[15] We report this information in intervals for confidentiality reasons

[16] We note that this is about a third of the theoretical maximum of $30 \times 117 = 3510$ observations that would be available if the 117 products were sold in all regions.

However, double- and triple-play products typically have higher average number of channels and bandwith than single-play products. Moreover, the average price of these bundles is typically smaller than the sum of single-play average prices. This fact raises the possibility that bundling may be motivated by potential cost savings from selling several products jointly. We test for this possibility below.

The 117 different products sold by firms are only a subset of the total possible products that firms could sell. Considering the total number of possible combinations of FV, the variants of bandwidth and number of channels in BB and TV services, respectively, there are a total of 478 products that firms could potentially sell. This restriction of product supply by firms impacts the total number of alternatives consumers can choose from.

Consumers choose among alternatives, i.e., combinations of products, possibly supplied by different firms. The concept of alternative does not coincide with the concept of service or a product offered by a firm. A product offered by a firm may be present in several choice alternatives. For example, the single-play product of fixed telephony offered by a given firm is typically present is several alternatives where FV service is provided. There are eight possible combinations of services, six possible types of bundles, and seven possible suppliers, with one, f_0, corresponding to the inexistence of a supplier. Taking into consideration the variants of bandwidth and number of channels in BB and TV services, respectively, as well as related products sold by firms, there are a total of $1,153$ alternatives, including the outside option of no service, that consumers can consider out of the 117 different products sold by firms. In practice, the actual number of alternatives available for a consumer depends on which market he is located, as product availability differs considerably by region. Table 5 provides a simplified illustration of some alternatives.

$[Table\ 5]$

5.2 Estimation

The estimation of differentiated product models is compositionally involved, due to the need to solve for unobserved product characteristics using market share equations, as in, e.g., Berry et al. (1995), Nevo (2000, 2001). However, recent developments in the estimation of these models can be used to simplify the estimation of our own model. We build on the estimation approaches of Su and Judd (2012) and Dube et al. (2012), where the step of solving for the vector of $\Delta\xi_{jt}$ is replaced with a constraint in the estimation problem. To this end, we reparametrize our demand model in a way similar to Nevo (2000, 2001). In what follows, we let $j = 1, ..., J_t^*$ denote the products that are actually observed in the data for each market $t = 1, ..., T$. The implied, available alternatives to consumers located in a market t are denoted $a = 0, 1, ..., A_t^*$.

Let J^* denote the total number of different products observed by the researcher across all markets in the data.[17] Let $\mathbf{b}$ be a $J^* \times 1$ column vector such that, for each $j = 1, ..., J^*$:

$$\mathbf{b}_j = X_j \beta + \xi_j. \tag{23}$$

Then, the mean utility specification in (9) simplifies to:

$$\delta_{at} = \sum_{j \in a} \left(\mathbf{b}_j - \alpha p_{jt} + \Delta \xi_{jt} \right). \tag{24}$$

We compute the integral in (17) via simulation. That is, using a total of n_S random draws from the multivariate standard Normal distribution $N(\mathbf{0}, \mathbf{I}_{K+1})$ and given values for the parameter vector $\zeta \equiv (\alpha, \mathbf{\Psi}, \mathbf{b}, \rho)$ and the unobserved demand shock $\boldsymbol{\Delta\xi}_t$, we approximate (17) by averaging the values of (16) across draws.[18] We denote this approximation as:

$$\hat{s}_{at}(X_t, \mathbf{p}_t, \xi_t, \Delta\xi_t) = \frac{1}{n_S} \sum_{m=1}^{n_S} s_{mat}(\boldsymbol{\theta}_m, \zeta, \boldsymbol{\Delta\xi}_t); \tag{25}$$

where random draws are indexed by $m = 1, ..., n_S$ and $s_{mat}(\boldsymbol{\theta}_m, \zeta, \boldsymbol{\Delta\xi}_t)$ is defined by the right-hand side of (16).

Let S_t denote the observed vector of market shares for products $j = 1, ..., J_t^*$ for market t, and define Γ_t as the $A_t^* \times J_t^*$ product inclusion matrix for market t as in (20). Denoting the vector stacking the predicted alternative shares (25) over all alternatives available in market t by $\hat{S}_t^{Alt}(\zeta, \boldsymbol{\Delta\xi}_t)$, we have for all $t = 1, ..., T$:

$$S_t = \Gamma_t' \hat{S}_t^{Alt}(\zeta, \boldsymbol{\Delta\xi}_t). \tag{26}$$

The equations system defined in (26) is used to define values for $\boldsymbol{\Delta\xi}_t$ given parameters ζ. As $\boldsymbol{\Delta\xi}_t$ consist of product demand shocks in market t, a natural way to estimate demand parameters is to form a GMM estimator where a set of moment conditions is satisfied as much as possible given some minimum distance criteria. Let Z_{jt} be a vector of instrumental variables that are mean-independent of $\Delta\xi_{jt}$ and where $\dim(Z_{jt}) \geq \dim(\zeta)$. Then, for some weighting matrix W, the GMM estimation problem is formalized as a Mathematical Program with Equilibrium Constraints (MPEC), as in Dube et al. (2012):

$$\min_{\zeta, \boldsymbol{\Delta\xi}_t} Q(\boldsymbol{\Delta\xi}_t, \zeta) = g(\boldsymbol{\Delta\xi}_t, \zeta)' W g(\boldsymbol{\Delta\xi}_t, \zeta) \tag{27}$$

$$s.t. \qquad S_t = \Gamma_t' \hat{S}_t^{Alt}(\zeta, \boldsymbol{\Delta\xi}_t), \quad \forall t = 1, ..., T$$

[17] In our sample we observed 117 products, i.e., $J^* = 117$.

[18] Recall that the random coefficients $\boldsymbol{\theta}_h = (\alpha_h, \boldsymbol{\beta}_h)$ are assumed to follow a multivariate Normal distribution with matrix $\mathbf{\Psi} = diag(\mathbf{\Psi}_p, \mathbf{\Psi}_X)$. The fact that a multivariate Normal distribution can be written as a linear transform of a multivariate standard Normal distribution allows us to keep the same draws from this distribution for each evaluation of the GMM objective function described below.

where the vector $g(\boldsymbol{\Delta\xi}_t, \zeta)$ is defined as:

$$g(\boldsymbol{\Delta\xi}_t, \zeta) = \frac{1}{T}\sum_{t=1}^{T}\frac{1}{J_t^*}\sum_{j=1}^{J_t^*}\Delta\xi_{jt} \cdot Z_{jt}. \tag{28}$$

The optimization problem (27) is solved using the stochastic, global optimization algorithm of named Covariance Matrix Adaptation Evolution Strategy (CMAES). This derivative-free algorithm is based on evaluation of candidate solutions picked at random using a multivariate Normal distribution centered at an initial guess, followed by selection of the candidates that yielded the lowest values for the objection function being minimized. The selected set of candidates is used to compute the mean and variance-covariance matrix of the multivariate Normal used to draw new candidate solutions, and the process is repeated until convergence. See Hansen (2006) for a discussion. We first solve (27) setting the weighting matrix to identity, i.e. $W = I$, and we form a new matrix W using the sample analog of the asymptotically efficient matrix evaluated at the solution just obtained. We resolve the problem using the new matrix W. Further updating of matrix W with new rounds of estimates did not lead to numerically significant changes in estimates.

We recover the vector of average taste parameters for observed characteristics β and the unobserved product quality vector $\boldsymbol{\xi}$ =$(\xi_1, ..., \xi_{J^*})$ using a GLS procedure similar to the one of Nevo (2001) applied to (23). Let $\hat{\mathbf{b}}_j$ be the estimates of the coefficients of the product dummy variables obtained after solving (27). The estimators for β and ξ are, respectively,

$$\hat{\boldsymbol{\beta}} = (X'V_bX)^{-1}X'V_b\hat{\mathbf{b}}, \tag{29}$$

$$\hat{\xi} = \hat{\mathbf{b}} - X\hat{\boldsymbol{\beta}}. \tag{30}$$

where V_b is the covariance matrix of $\hat{\mathbf{b}}$.

Standard errors for demand model estimates must be corrected for errors due to consumer sampling process and to integration-by-simulation process. That is, we must account for the fact that we observe estimated rather than actual product market shares and that simulation draws are the same for all observations in a market. See Berry et al. (1995) for a discussion. We correct for these errors in standard deviation calculation by resorting to nonparametric bootstrap methods. That is, we resample the data points within each market with replacement and resolve (27) using this artificial sample. This process is repeated N times, and standard errors of model estimated are computed as the standard deviations of the N solutions to (27) obtained with the N artificial samples. To speed the computations of the solutions to each of the N problems, we take the original solution to (27) as the initial guess and then minimize (27) with each artificial sample using a Newton-Raphson algorithm. Standard error estimates of $\hat{\mathbf{b}}$ are also used to craft its covariance matrix V_b necessary to run the GLS regression in (29). We obtain standard errors for $\hat{\boldsymbol{\beta}}$ by running the GLS regression in (29) for each bootstrap sample and then take the standard deviation of the N bootstrap solutions.

We are interested on testing for synergies in bundle marginal costs. To this end we use first-order conditions in prices to identify marginal costs. After deleting the rows and columns pertaining products that firms do not commercialize in market t, we infer from (5) that the vector of marginal costs of products commercialized in that market, $\mathbf{c}_t$, is given by:

$$\mathbf{c}_t = \mathbf{p}_t - (\mathbf{H} * \mathbf{\Delta})^{-1}\mathbf{s}_t(\mathbf{p}_t, \mathbf{d}_t) \tag{31}$$

We use the marginal cost data obtained using (31) to run OLS regressions akin to synergy testing, discussed below.

6 Results

In this Section we present the estimaton results of our demand model using the product/region data described in Section 5.1. We discuss alternative specifications in regards to consumer heterogeneity and instrumental variable set being used. The variables considered for product observed characteristics vector X_j include service dummy variables the logarithm of number of offered channels, the logarithm of bandwidth (measured in in Mbps) and both double- and triple-play bundle dummy variables.[19] We exclude service dummy for TV to avoid colinearity with other dummy variables considered in the regression. Standard errors of estimates were computed using the bootstrap procedure described in the previous section for $N = 500$ bootstrap samples.

To estimate our model we need a vector of variables Z_{jt} that is independent of $\Delta\xi_{jt}$. We consider several possible instruments that have been considered in the differentiated product demand literature. Berry et al. (1995) propose using other product characteristics in X_j as instruments, as well as sums of other products' characteristics sold by the same firm and sums of rival firms' product characteristics. While using X_j in the instrument set is not possible since it is collinear with product dummies, sums of firm's own and rival products can be considered in the instrument set for solving (27). Bresnahan et al. (1997) extend this instrument set further by considering charactristics sums within nests. Along with market- and product dummies, this is one of the main instrumental variables set we consider in estimation.

Nevo (2001) and Hausman (1996) propose for the IV vector Z_{jt} the prices for product j at all regions other than t, provided that a certain set of assumptions apply. As those assumptions cannot be directly tested, we resorted to estimate the model using regional average prices. We considered a NUTS II set of markets as a region, and we formed instruments by computing the average price of a product in that region, excluding the price at the

[19] To be exact, for continuous attributes such as channels or bandwidth number we considered the function $\log(x+1)$, where is the quantity of interest. This ensures decreasing marginal utility in the attribute while avoiding a log of zero when the product has neither TV nor BB service.

market where the product is sold[20]. We then compare results using alternative instrument sets.

Before examining the more general versions of the model, it is important to examine estimation results when certain model features are not considered. Table 6 presents the estimation results for product demand under no consumer heterogeneity (i.e. standard deviations of taste parameters are set to zero so that all consumers have same utility function coefficients). The first two columns displays the particular case of the PD-GEV model when nest parameters are set to one, i.e., the logit model.[21] The first column presents logit model estimates under the (implausible) assumption that prices are not correlated with the product demand shock, while the second considers the instruments proposed by Berry et al. (1995) and Bresnahan et al. (1997) are used to control for that possible source of endogeneity. While the simple logit specification for alternative choice leads to a negative price coefficient, as expected, it is nearly half, in absolute value, to its analog of the second column. Moreover, several taste coefficients have unexpected signs (e.g. log of bandwidth, log of channels). This pattern is somewhat similar to other examples where not controling for endogeneity leads to implausible estimates (e.g. as in Berry, Levinsohn and Pakes 1995). We conduct a similar exercise with the PD-GEV specification, in order to have a feel over possible benefit over the logit specification. While similar comments apply in regards to instrumenting for price, estimating nest parameters rather than setting them to one seems to be a nontrivial improvement. First, the nest parameters are in general significantly different from one. Second, the price coefficient increased (in absolute value) both with and without instrumenting. This suggests that controling for possible correlations across alternative shocks is important on identifying price sensitivity. This situation is somewhat similar to the results of Bresnahan et al. (1997), where they find that the PD-GEV yields higher (in absolute value) price coefficients that two-level, nested logit specifications.

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[*Table* 6]

Tables 7 and 8 report model of the full demand model under different instrument sets and nest parameter assumptions. The first three columns contemplate the mixed-logit case where nest parameters are set to one. The first two columns compare estimates with and without instrumenting for price. The results indicate that allowing for mixing does not mitigate the price endogeneity problem, as the coefficient for price using the instruments of Berry et al. (1995) and Bresnahan et al. (1997) is nearly double, in absolute value, of the one without controling for endogenous prices. The third column reports estimates using average regional prices as instruments, e.g. as in Nevo (2001) and Hausman (1996). While

[20] As some products are supplied only in certain regions, we considered the fit of regressing prices on X and regional dummies whenever the price of a product in a region was unavailable.

[21] In contrast to most of the empirical literature in differentiated product demand, we cannot run OLS and 2SLS estimation. The reason is that product market shares do not add up to one as a result of households choosing *alternatives,* not just *products.*

the price coefficient is bigger, in absolute value, than the one of first column, it is smaller than the one of the second column. Moreover, some taste parameters have unuintive signs despite being statistically insignificant (e.g. log of bandwidth, FV dummy). This evidence suggest that some of the assumptions validating average regional prices as instruments may not be met. The fourth fifth and sixth columns are pertain the mixed-PDGEV specification results using instrument sets in the same order as the preceding three columns. The results confirm that the instruments of Berry et al. (1995) and Bresnahan et al. (1997) yield more plausible estimates that using alternative instrument sets. Moreover, several of the standard deviations of the taste parameter distribution are significant, suggesting that consumer heterogeneity is an important feature of this market. Thus we select estimates of the fifth column, i.e., we will consider estimates of the mixed-PDGEV model using Berry et al. (1995) and Bresnahan et al. (1997) in all model simulations described below.

[*Table* 7]

[*Table* 8]

A potential reason for the practice of bundling may be the presence of potential cost savings from selling several products in bundles. Indeed several bundles sell at a discount compared to the sum of their individual prices, but that may or may not be cost-motivated. We use model estimates from the fifth column of Tables 7 and 8 to recover marginal costs using equation (31) and test for cost synergies. Letting $\hat{c}_{jt}$ denote our estimate of marginal cost of product j in market t, and Y_j a 7×1 vector of single-, double- and tripleplay dummies[22], we run the auxiliar regression

$$\hat{c}_{jt} = Y_j \beta_Y + A_{jt} \beta_A + \varepsilon_{jt} \tag{32}$$

where A_{jt} is a vector of shifters (e.g. channels, bandwidth, market dummies).

.Table 9 presents estimates of (32) for different regressor sets in vector A_{jt}. For each regression, we test for cost synergies by forming linear hypothesis on β_Y of the form

$$H_0 \quad : \quad R \cdot \beta_Y \geq 0 \tag{33}$$

$$H_A \quad : \quad R \cdot \beta_Y < 0 \tag{34}$$

[22] Recall that there are seven types of products: single-play FV, single-play TV, single-play BB, single-play FV+TV, single-play FV+BB, single-play TV+BB, tripleplay.

where R is a vector defining the restriction. For example, $R = (-1, -1, 0, 1, 0, 0)$ implies an alternative hypothesis $\beta_{Y,4} < \beta_{Y,1} + \beta_{Y,2}$, i.e., that the average cost of double-play FV+TV is small than the cost of its individual components (i.e. single-play FV plus single-play TV). It follows from standard OLS results that $R \cdot \beta_Y$ is asymptotically Normal and centered at zero.[23] We use this result to test every possible combination of cost synergies envolving single-, double- and tripleplay. Except for the single case of the regression with only product type dummies and for the alternative hypothesis that triple-play costs less thant the sum of single-play BB and double-play FV+TV, we do not reject the null hypothesis. This result still applies even if we consider strict equality in the null hypothesis. Thus, while not rejecting the null hypothesis does not imply it is true, these results suggest that cost synergies are not a likely driver of bundling behavior.

[*Table* 9]

7 Simulation

The model described in Section 4 allows us to simulate pricing and bundling behavior for different scenarios. In this paper we focus attention on two counterfactuals. First, we examine market equilibrium changes when the standard deviation of the price coefficient of demand is doubled. The interest of this exercise is motivated by theoretical results in the bundling literature, where it is predicted that higher consumer heterogeneity leads to higher bundling frequency as a means to replicate price discrimination. Second, we examine how is bundling behavior sensitive to changes in market structure. In particular, we focus on simulating behavior of a monopolist firm and compare prices, bundling behavior and product characteristics with a duopoly case, where the firm competes against a clone of itself (i.e. with a firm with the same set of products). Again this exercise is motivated by its theoretical interest. The theory relating bundling and market structure is not consensual on whether bundling increases or decreases the level of competition. We use the estimated model to assess what happens to market equilibrium when we move from monopoly to duopoly. For this exercise we pick one of the top firms in this market.[24]

We focus attention on only one of the 30 markets used in estimation. For our simulations, the chosen market was the Greater Lisbon area. As firms only sell some of the potential 478 products that could be sold in each market, we need to calibrate values for their marginal costs and unobserved characteristics. We use the estimated marginal cost function with market dummies to assign marginal costs for products in the Geater Lisbon market. We

[23] Its variance-covariance matrix is $\sigma^2 R(X'X)^{-1}R'$, where X is matrix of regressors and σ^2 is the variance of regression residuals.

[24] For confidentiality concerns we avoid identifying the firm in question.

use sample averages of ξ_j and $\triangle\xi_{jt}$ by product type and market to quantify unobserved characteristic on products not sold in the data. After replacing unknown parameters in demand and supply equations, we solve for the new equilibrium by minimizing the quadratic distance between right- and left-hand side of equations (5) and (3). Formally, define the multivariate function

$$K(\mathbf{p}_t, \mathbf{d}_t) = \left[\begin{array}{c} \mathbf{d}_t * \mathbf{s}_t(\mathbf{p}_t, \mathbf{d}_t) - (\mathbf{H} * \mathbf{\Delta}) \left[\mathbf{d}_t * (\mathbf{p}_t - \mathbf{c}_t) \right] \\ \Pi_{ft}(\mathbf{p}_t, \mathbf{d}_t) - \max \left\{ \Pi_{ft}(\mathbf{p}_t, \mathbf{d}_{-jt}, 0), \Pi_{ft}(\mathbf{p}_t, \mathbf{d}_{-jt}, 1) \right\}, \quad j = 1, ..., J_t \end{array} \right] \tag{35}$$

The equilibrium in market t can be computed by solving the minimum distance problem

$$\min_{(\mathbf{p}_t, \mathbf{d}_t)} \| K(\mathbf{p}_t, \mathbf{d}_t) \| \tag{36}$$

Since each $\mathbf{d}_t$ is a vector of bynary variables and each $\mathbf{p}_t$ is a vector of continuous variables, the problem defined in (36) is a mixed-integer program. To solve this problem we again resort to the stochastic global optimization algorithm CMAES (Hansen 2006), which is capable to handle problems with both discrete and continuous controls.

Table 10 presents simulation results for our first counterfactual. We partition the 478 possible products into the seven possible types of single-double- and tripleplay. The percentage of commerxialized products refers to what percentage of those product types is actually sold in the market (i.e. it is the frequency of the discrete control d_{jt} within that class). The results indicate that, in response to doubling the standard deviation of the price sensitivity parameter, more products are introduced in the market except for single-play FV. The decision to sell more products is more marked for double- and triple-play products. However, average equilibrium product prices seem not to decrease substantially in response to having additional products sold in the market. This is partially explained by the increase in the average number of channels and average bandwidth of sold products. These results are consistent with the argument that higher consumer heterogeneity leads to higher bundling frequency, yet firms tend to provide "better" products in the form of more channels and more bandwidth in order to attract customers. Thus, in addition to competing in prices, firms seem also to compete in quality through product commercialization decisions.[25]

$[Table\ 10]$

[25] Note that if a firm owns two products of the same type that just differ in either the number of channels or bandwidth size (or both), the firm de facto increases product quality by selling the product with highest channels and/or bandwidth while not selling the product with lower attribute values.

We present our monopoly and duopoly simulation results on Table 11. The interpretation of each row is analogous as in the previous experiment, except that product sale decisions consider only products that the chosen firm (and its clone) can potentially sell in this market. For example, the monopolist can sell at most one single-play FV, while in duopoly we can observe at most two single-play FVs (one for each firm). Except for single-play FV and the bundle FV+TV, the monopolist has low rates of product introduction in the market. However, the percentage of commercialized products is strictly positive, meaning that the monopolist does bundling as a means to profit maximization. Bundling behavior intensifies as one more similar firm enters the market. As expected, average equilibrium prices decrease, yet we also observe increases in the average number of channels and average bandwidth size. Thus firms do compete not only in prices, but also in the quality dimension via product commercialization decisions. To our knowledge, this dimension of competition is not present in most of the bundling literature, where attention is focuses primarily in pricing decisions. Our results indicate that dimensions other than price should be important on characterizing markets where firms may engage in bundling behavior.

[*Table* 11]

8 Conclusion

When firms offering several services supply bundles of products, conventional models of supply and demand for differentiated products cannot be directly used to predict market equilibria. In this paper we provide a framework that deals with this problem. We model consumer demand for multiproduct alternatives that contemplate different services as a means to identify consumer interest for bundles of products. Our framework is estimable and akin to simulations of bundling behavior. We apply the MPEC estimation approach of Dube, Fox and Su (2012) to estimate our model by applying a GMM estimator to a data set from the Portuguese triple-play market. Our estimates indicate that consumer heterogeneity and correlation between preferences for alternatives are important features of demand for triple-play products. Moreover, marginal cost data implied by our model is consistent with lack of cost synergies. Instead, our simulation using the model indicates that consumer heterogeneity and strategic firm behavior are more plausible sources of bundling behavior. Product commercialization decisions prove also to be an important control along with price on bundling behavior. This extra firm decision is absent from most theoretical bundling models, yet our results indicate it may be important in practice. Our framework can be applied to other settings where firms combine products to form bundles.

While our approach allows the researcher to estimate demand and supply for bundles when only product information - and not assortment choices - are available, it has some limitations. First, our extension of the PD-GEV framework of Bresnahan et al. (1997)

deals with the issue of correlation across alternatives due to containing similar products, yet its potential to deal with a many alternatives may be limited in some applications. For example, if there is a very large number of product variants within a certain category, or if the number of industries to consider is excessive, the number of possible alternatives may be too large to be computationally tractable. Second, the problem of solving for equilibrium where the researcher solves for both prices and decisions of commercialization may be hard to implement in practice. Recent developments in large-scale optimization and its application to industrial organization problems, e.g. Dube, Fox and Su (2012), Su and Judd (2012), may deal with this problem to a large extent.

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A Tables

Table 1: Market shares

	Fixed voice		Pay-TV		Broadband	
	2008	2009	2008	2009	2008	2009
PT	65.7%	61.6%	13.6%	23.0%	41.6%	44.5%
ZON	4.4%	10.2%	72.3%	64.4%	31.3%	32.2%
Optimus	16.3%	14.5%	0.5%	1.0%	12.5%	9.2%
Vodafone	5.1%	6.1%	-	0.3%	2.8%	3.9%
Cabovisão	3.3%	3.6%	12.4%	10.2%	9.3%	8.0%
AR Telecom	1.7%	1.4%	1.0%	0.9%	1.5%	1.4%
Others	0.7%	0.5%	0.1%	0.1%	1.0%	0.8%

Market share in terms of subscribers, except for fixed telephony which is in terms of traffic. Source: ICP-ANACOM (Relatórios trimestrais)

Table 2: Products - Notation

Services

N	Notation	Description
1	000	no serv.
2	100	FV
3	010	TV
4	001	BB
5	110	FV+TV
6	101	FV+BB
7	011	TV+BB
8	111	FV+TV+BB

Bundles

N	Notation	Description
1	p000	no serv.
2	no b	No bundle - Single play
3	p110	Double play FV+TV
4	p101	Double play FV+BB
5	p011	Double play TV+BB
6	p111	Triple play FV+TV+BB

Firms

N	Notation
1	f_0
2	f_1
3	f_2
4	f_3
5	f_4
6	f_5
7	f_6

Table 3: Services vs. bundles

Services	p000	no b	p110	p101	p011	p111	Total
	Bundles						
000	[26-28%]	0%	0%	0%	0%	0%	[26-28%]
100	0%	[14-16%]	0%	0%	0%	0%	[14-16%]
010	0%	[10-12%]	0%	0%	0%	0%	[10-12%]
001	0%	[0-2%]	0%	0%	0%	0%	[0-2%]
110	0%	[4-6%]	[4-6%]	0%	0%	0%	[10-12%]
101	0%	[0-2%]	0%	[4-6%]	0%	0%	[4-6%]
011	0%	[0-2%]	0%	0%	[4-6%]	0%	[6-8%]
111	0%	[0-2%]	0%	[2-4%]	[0-2%]	[16-18%]	[22-24%]
Total	[26-28%]	[36-38%]	[4-6%]	[6-8%]	[4-6%]	[16-18%]	100%

Distribution of services consumed per type of bundle, 2009. Source: ICP-ANACOM, "Inquï£¡rito ao consumidor"

Table 4: Average characteristics by product type

Product	Price	Share	channels	bandwidth
FV only	9.245	0.136	0	0
TV only	36.381	0.048	70.020	0
BB only	26.890	0.019	0	5.823
FV+TV	50.981	0.008	65.309	0
FV+BB	24.345	0.010	0	18.018
TV+BB	55.742	0.011	109.523	15
FV+TV+BB	56.362	0.013	89.266	18.453

Table 5: Alternatives - Simplified Examples

N	Services	Bundles	S. FV	S. TV	S. BB	Description
0	000	p000				No services
1	100	no b	f_1			Fixed voice from f_1
2	111	p111	f_2	f_2	f_2	Triple-play from f_2
3	010	no b		f_2		Pay-TV from f_2
4	111	p111	f_1	f_1	f_1	Triple-play from f_1
5	101	p101	f_4		f_4	Double play (FV+BB) from f_4
6	110	no b	f_1	f_2		Fixed voice from f_1 + Pay-TV from f_2
...						

S. FV - supplier of FV; S. TV - supplier of TV; S. BB - supplier of BB

Table 6: Demand Estimates - models without heterogeneity

	Models			
	Logit	IV Logit	PDGEV	IV-PDGEV
const.	-0,450	-2,503	-0,022	-2,355
	(0,169)	(0,889)	(0,115)	(0,735)
price coef.	-0,038	-0,070	-0,074	-0,099
	(0,011)	(0,007)	(0,018)	(0,031)
p100	-1,535	0,671	-3,392	0,671
	(0,892)	(0,129)	(0,996)	(0,232)
p001	-5,457	1,659	-2,926	1,659
	(1,992)	(0,773)	(1,315)	(0,563)
p110	0,622	1,680	1,036	1,680
	(0,108)	(0,946)	(0,333)	(0,231)
p101	2,512	1,964	0,782	1,964
	(0,727)	(0,451)	(0,222)	(0,199)
p011	4,235	0,147	3,311	0,079
	(1,012)	(0,024)	(1,088)	(0,022)
p111	6,004	1,287	5,139	1,285
	(0,991)	(0,556	(1,013	(0,341
log(channels+1)	-0,343	0,452	-0,390	0,452
	(0,331)	(0,200)	(0,223)	(0,210)
log(bandwidth+1)	-0,766	0,788	-0,669	0,788
	(0,553	(0,316	(0,445	(0,225)
ρ_{FV}			0,623	0,530
			(0,112	(0,201)
ρ_{TV}			0,452	0,496
			(0,023)	(0,103)
ρ_{BB}			0,223	0,509
			(0,099)	(0,113)
Numb. Obs.	1083	1083	1083	1083
GMM Obj	7,098	6,956	6,112	6,004

Table 7: Demand Estimates - Full Model

		Models					
		(1)	(2)	(3)	(4)	(5)	(6)
Mean Parameters	constant.	-0,020	-2.417	-1.124	0.932	-2.353	-1.126
		(0,022)	(0.923)	(0.881)	(0.702)	(0.992)	(0.298)
	price coef.	-0,036	-0.078	-0.043	-0.067	-0.167	-0.085
		(0.010)	(0.019)	(0.005)	(0.040)	(0.047)	(0.022)
	FV	-3.397	0.671	-0.638	-2.421	0.671	-1.017
		(1.911)	(0.100)	(0.406)	(1.100)	(0.067)	(0.081)
	BB	-3.716	1.659	-1.028	-3.040	1.659	-1.388
		(1.890)	(0.211)	(0.500)	(1.112)	(0.600)	(0.999)
	p110	2.736	1.680	0.684	0.371	1.680	-0.417
		(1.012)	(1.012)	(0.561)	(0.145)	(0.799)	(0.335)
	p101	4.501	1.964	1.269	0.841	1.964	0.708
		(1.099)	(0.778)	(0.998)	(0.512)	(0.893)	(0.690)
	p011	5.342)	0.130	2.624	3.624	0.079	-0.061
		(1.222	(0.092)	(1.113)	(0.444)	(0.045)	(0.111)
	p111	7.999	1.274	3.665	4.473	1.285	2.665
		(0.900)	(0.699)	(0.799)	(1.668)	(0.456)	(0.954)
	log(channels+1)	-0.576	0.452	0.093	-0.561	0,452	-0.092
		(0.559)	(0.301)	(0.111)	(0.488)	(0.311)	(0.140)
	log(bandwidth+1)	-0.333	0.788	-1.210	-1.048	0.788	-0.638
		(0.402)	(0.210)	(0.709)	(0.665)	(0.301)	(0.889)
Nest Parameters	ρ_{FV}				0.530	0.633	0.623
					(0.110)	(0.200)	(0.234)
	ρ_{TV}				0.496	0.664	0.711
					(0.291)	(0.200)	(0.441)
	ρ_{BB}				0.509	0.685	0.699
					(0.199	(0.334)	(0.302)

Table 8: Demand Estimates - Full Model - Continued

		Models					
		(1)	(2)	(3)	(4)	(5)	(6)
STD Parameters	constant.	1.675	1.319	1.878	1.878	1.299	1.878
		(2.098	(1.005)	(1.335)	(0.991)	(1.509)	(1.112)
	price.	0.029	0.024	0.014	0.056	0.014	0.067
		(0.020)	(0.010)	(0.012)	(0.045)	(0.066)	(0.022)
	FV	1.529	1.404	1.541	1.541	1.404	1.542
		(1.339)	(1.112)	(1.711)	(2.010)	(1.130)	(0.789)
	BB	1.538	1.205	1.538	1.624	1.205	1.822
		(0.991)	(0.890)	(1.133)	(0.451)	(0.581)	(0.901)
	p110	1.450	1.137	1.450	1.450	1.137	1.452
		(1.194)	(0.982)	(1.222)	(1.023)	(2.200)	(2.011)
	p101	1.625	1.274	1.625	1.626	1.274	1.626
		(0.400)	(0.509)	(0.668)	(0.333)	(0.411)	(0.550)
	p011	1.667	1.317	1.667	1.668	1.307	1.784)
		(0.892)	(0.694)	(0.801)	(0.901)	(0.500)	(0.367)
	p111	2.929	2.295	2.929	2.929	2.295	2.929
		(1.123)	(0.967)	(1.078)	(0.460)	(0.710)	(0.666)
	log(channels+1)	0.330	0.390	0.411	0.471	0.390	0.412
		(0.133)	(0.199)	(0.167)	(0.113)	(0.089)	(0.056)
	log(bandwidth+1)	0.286	0.230	0.346	0.361	0.218	0.347
		(0.128)	(0.190)	(0.100)	(0.126)	(0.066)	(0.046)
Num. Obs.		1083	1083	1083	1083	1083	1083
GMM Obj		6.872	6.432	6.773	6.512	5.812	5.991

Table 9: Marginal Cost Regressions

	Regression #1		Regression #2		Regression #3	
Variable	Coefficient	Std. Error	Coefficient	Std. Error	Coefficient	Std. Error
mc p100	3.475	2.136	9.472	3.798	3.387	5.453
mc p010	30.370	1.982	10.740	3.814	4.207	5.537
mc p001	20.981	1.874	23.252	3.748	16.585	5.533
mc p110	42.749	1.305	22.974	3.740	16.775	5.453
mc p101	19.702	1.891	23.052	3.598	16.628	5.396
mc p011	48.262	1.891	28.356	3.897	22.288	5.574
mc p111	50.073	1.102	31.390	3.686	25.252	5.410
channels			0.152	0.017	0.151	0.017
bandwidth			0.208	0.036	0.196	0.037
f_1			-0.663	3.630	1.212	3.755
f_2			-5.526	3.859	-4.249	3.964
f_3			53.834	3.762	55.434	3.903
f_4			-7.535	3.742	-6.196	3.854
f_5			-10.028	3.670	-8.194	3.797
time dummies					yes	
Adj R-squared	0.80		0.93		0.93	

Table 10: Simulation Results - Increased Consumer Heterogeneity

	% of products offered		average price		average channels		average bandwidth	
	observed	2x std.(α)	observed	2x std.(α)	observed	2x std.(α)	observed	2x std.(α)
p100	50.0%	50.0%	8.97	8.75	0	0	0	0
p010	20.8%	25.0%	29.78	28.60	67	62	0	0
p001	25.0%	30.6%	34.52	33.55	0	0	23	21
p110	56.5%	69.6%	56.48	55.93	68	0	0	0
p101	36.1%	52.8%	27.80	27.22	0	0	23	24
p011	5.7%	11.9%	51.59	51.41	68	71	25	27
p111	16.9%	21.5%	55.42	53.60	68	75	25	27

Table 11: Simulation Results - Monopoly vs Duopoly

	% of Products Offered		Average Price (€)		Average Channels		Average Bandwidth	
	Monopoly	Duopoly	Monopoly	Duopoly	Monopoly	Duopoly	Monopoly	Duopoly
p100	100.0%	100.0%	22.2	17.9	0	0	0	0
p010	10.0%	15.0%	34.6	31.5	25	38	0	0
p001	18.2%	0,22.7%	30.3	27.6	0	0	4	11
p110	40.0%	45.0%	51.2	45.0	45	70	0	0
p101	18.2%	31.8%	26.8	24.9	0	0	12	19
p011	2.0%	5.0%	56.8	48.6	55	69	18	29
p111	3.0%	5.5%	69.4	55.0	53	70	21	33