Mobile Call Termination and Collusion under Asymmetry

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Abstract

This paper looks at duopolistic competition in the Telecommunications industry with non-linear tariffs and network based price discrimination. We employ the standard Hotelling framework of horizontal product differentiation but allow for differentiation in a second dimension. Modulo locations consumers may have different demand elasticities with respect to the two networks which can capture, for example, differences in network histories. The implications of these asymmetries on the possibility to sustain collusion are investigated under alternative access pricing regimes.

1 Introduction

The literature on network interconnection and pricing strategies in the Telecommunications industry originating in the work of Armstrong (1998) and Laffont, Rey, and Tirole (LRT 1998a,b) has generically assumed that competition takes place between *symmetric* networks.

This assumption of symmetry of the two networks in previous models is a most welcome simplifying device to keep the analysis of the pricing vectors that form the Nash equilibrium of the game tractable. However, in most cases of regulatory concern it is a later entrant that competes against an incumbent with an established market share and possibly also against substantial switching costs. Thus the assumption, being at the source of various "neutrality results", see LRT (1998a) and Dessein (2003) seems to be unfortunate.

Previous research in *asymmetric* telecommunication environments is still scarce. Carter & Wright (2003) show that firms, given that access charges have to be chosen

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reciprocally (i.e. symmetrically), may prefer them to be set at cost if size differences are pronounced. Peitz (2005) investigates the issue of asymmetric regulation but focuses on entry and consumer surplus. These two approaches are extended by Stühmeier (2012) to asymmetric termination costs. Hoernig (2007) finds evidence that larger firms will tend to have a larger price differential between its on- and off-net prices but does not model access charges explicitly.

As shown in Behringer (2009) one can indeed find *non-reciprocal* equilibrium access charges with a positive markup on termination cost as observed in regulatory practice by simultaneously assuming that such charges are chosen *non-cooperatively* and that networks are potentially *asymmetric.* This gap in the literature has been noted as early as in Armstrong (2002, p.373)) and Geoffron & Wang (2008) employ the same modelling of asymmetry to investigate the effects of calling clubs. Alternative explanations for positive markups are provided in Armstrong & Wright (2009) and Jullien, Rey, & Sand-Zantman (2010). A collective volume dedicated to the issue of asymmetries in mobile markets in order to increase realism as demanded in a study for the European Commission (see Tera (2009), p.133) is Benzoni & Geoffron (2007).

The issue of *collusion* has been present in the analysis of the Telecommunication industry from the very beginning. For an introductory overview see Peitz et. al. (2004). However all these investigations focus on the effect of access charges on the resulting retail price components only. An exception to this is the more recent work of Höffler (2009) who looks at collusion in the classical way of an infinitely repeated Bertrand competition setting with heterogenous consumers. Again, however, firms are assumed to be symmetric.

One of the few papers that have joined the issues of *asymmetric firms* and *incentives to collude* is Baranes & Poudou (2009). It has long been consensus that collusion is easier to sustain among symmetric firms. Their model allows for differing price sensitivities of consumer demands (e.g. resulting from switching costs) and access charges and they find that symmetry in access regulation may actually hinder collusion. The model employs a differentiated duopoly framework which the present paper extends to the standard horizontal differentiation Hotelling setup as commonly used for the Telecommunications industry allowing for two-part tariffs and network based price discrimination.

Today regulators intend to reduce the level of asymmetric regulation of access charges according to a *glide path.* This is especially true in case of fixed and mobile phone termination rates (see European Commission (2009)) and for the current discussions on the implementation of 'bill-and-keep' regimes. Thus the relevance of extending the findings in Baranes & Poudou (2009) to the particularities of the

Telecommunications industry follows naturally.

Section 2 presents the model in which the competitive, the collusive, and the deviation outcome are laid out. Section 4 investigates how the critical discount factors are affected by both demand asymmetries and access pricing regimes. Section 6 concludes. Proofs are relegated to an Appendix.

2 Model setup

The following model uses the setting of the network competition models of Laffont, Rey, & Tirole (1998) with duopolistic competition in two-part tariffs, on-net and off-net price discrimination, and balanced calling patterns. As Carter and Wright (1999, 2003), we allow explicitely for exogeneous asymmetry between networks. However, we consider that the asymmetry is directly related to the demand for calls. That is, asymmetry doesn't lie on the fixed utility but directly affects the volume of calls from consumers. We then consider the question of collusion sustainability in this industry focusing on the role of the demand asymmetry.

Demand asymmetry. To model demand asymmetry between networks, we consider consumer demand for calls is given by $q(p, \eta)$, where p is the unit price and η is some parameter assumed to encompass the asymmetry. This parameter can represent the elasticity of demand or measure the size of the demand (or network), or can be generalized to any other type of heterogeneity with relative importance for each network respectively. In our duopoly setting we denote η_i as the parameter for network i, with $i = 1, 2$. Considering network i, we assume that the asymmetry parameter exceeds a given level, $\eta_i \geq \eta_0$. Networks are then symmetric when $\eta_i = \eta_0$ ($\forall i = 1, 2$) and asymmetric otherwise. The indirect utility $v(p, \eta)$ a consumer gets from consuming q unit of calls is given by:

$$v(p, \eta) \equiv \int_p^\infty q(\zeta, \eta) d\zeta = u(q(p, \eta), \eta) - pq(p, \eta)$$

where $u(q, \eta)$ represents the gross utility from making q calls.

We consider standard assumptions both on demand and indirect utility[1]: $q_1(p, \eta) < 0$ and $v_1(p, \eta) = -q(p, \eta) < 0$, for all (p, η). That is, both quantity and indirect utility are decreasing functions of unit price. Moreover, we maintain the following assumption:

Assumption. *For all (p, η), demand and indirect utility functions either satisfy: A.1)* $\operatorname{sgn}(q_2(p, \eta)) = \operatorname{sgn}(v_2(p, \eta)) < 0$, *or, A.2)* $\operatorname{sgn}(q_2(p, \eta)) = \operatorname{sgn}(v_2(p, \eta)) \geq 0$

[1] Hereafter, lower indices denote the position of the argument of the function for which the partial derivative is taken.

Assumptions *A.1)* and *A.2)* deserve more comments. For a given price the effects of asymmetries on demand and indirect utility are coinciding. Assumption *A.1)* (resp. *A.2)*) imply that if the asymmetry has a decreasing impact on demand it will also decrease the consumer indirect utility and vice versa. Assume $\eta_1 \geq \eta_2$, then Assumption *A.1)* implies that both demand and indirect utility derived from network 1 are always lower than those derived from network 2. The reverse applies for Assumption *A.2)*.

To illustrate Assumption 2 consider the following examples:

Example 1 (isoelastic demand): Consider the isoelastic demand function given by $q(p,\eta) = A(\eta)\, p^{-\epsilon(\eta)}$, where $A(\eta) > 0$ is the size of demand (or network) and $\epsilon(\eta) > 1$ is the elasticity of demand. The indirect utility is then $v(p,\eta) = \frac{A(\eta)p^{1-\epsilon(\eta)}}{\epsilon(\eta)-1}$. First consider that the asymmetry weighs fully on the demand size, i.e. $A(\eta) = \eta$ and $\epsilon(\eta) = \epsilon$. Then one can see that $q_2(p,\eta) = p^{-\epsilon} > 0$ and $v_2(p,\eta) = \frac{p^{1-\epsilon}}{\epsilon-1} > 0$. Now, consider asymmetry weighs fully on the demand elasticity, i.e. $A(\eta) = A$ and $\epsilon(\eta) = \eta$. Then $q_2(p,\eta) = -\ln(p)q(p,\eta) < 0$ and $v_2(p,\eta) = -\frac{(1+(\eta-1)\ln(p))v(p,\eta)}{(\eta-1)} < 0$, for a given price $p > 1$.

Example 2 (linear demand): Consider the linear demand function given by $q(p,\eta) = A(\eta) - B(\eta)\, p$, where $A(\eta) > 0$ is the size of demand (or network) and $B(\eta) > 0$, a slope parameter that increases[2] the elasticity of demand for a given price. The indirect utility function is then $v(p,\eta) = \frac{1}{2B(\eta)}q(p,\eta)^2$. Hence, if we consider asymmetry weighs fully on the demand size, i.e. $A(\eta) = \eta$ and $B(\eta) = B$, then one can see that $q_2(p,\eta) = 1 > 0$ and $v_2(p,\eta) = q(p,\eta)/B > 0$. If asymmetry weighs fully on the elasticity proxy parameter B, i.e. $A(\eta) = A$ and $B(\eta) = \eta$, then both demand and indirect utility are decreasing function of η, $q_2(p,\eta) = -p < 0$ and $v_2(p,\eta) = -\frac{1}{2\eta^2}q(p,\eta)(A + \eta p) < 0$.

Network market shares and profits. We consider competition between two networks, $i = 1, 2$, located at the opposite ends of a Hotelling unit line, with network 1 located at $x = 0$ and network 2 at $x = 1$. Consumers are assumed to be uniformly distributed on the unit line and the transportation cost is denoted by $\theta > 0$ per unit. Both networks offer a two-part tariff to consumers including the fixed fee, f_i, the on-net unit price, p_i, and the off-net unit price, $\hat{p}_i$. Hence, under a balanced calling pattern, a consumer purchasing from network i obtains a net surplus given by

$$w_i = \alpha_i v(p_i, \eta_i) + (1 - \alpha_i)v(\hat{p}_i, \eta_i) - f_i \tag{1}$$

[2]Clearly $B(\eta)$ is not the elasticity here, it writes $\epsilon = B(\eta)\, p/(q(p,\eta)^2$. However, increasing $B(\eta)$ increases the demand elasticity since $d\epsilon/dB(\eta) = A(\eta)\, p/q(p,\eta)^2 > 0$.

where α_i denotes the market share of network i.

Given the unit transportation cost θ, a consumer who is identified by his location x gets an overall utility $w_1 - \theta x$, if he joins network 1, and $w_2 - \theta(1-x)$ if he joins network 2. The marginal consumer between network 1 and 2 is defined by $\hat{x} \equiv (\theta + w_1 - w_2)/2\theta$.

Each network is bearing a fixed cost normalized to 0, and the same marginal costs c_0 at the originating or terminating end of each call. For each unit of an off-net call from network j to network i, network j pays the termination fee a_i. Then, the per unit cost of an off-net call is $c_0 + a_i$ whereas the per unit cost of an on-net call is $2c_0$.

We will restrict attention to market conditions for which the market is fully covered by the networks. This will be the case in particular when networks are very similar (η_1 close to η_2) and termination fees are cost-based (a_1 and a_2 close to c_0). To ensure this formally, we assume that networks are moderately differentiated so that:

$$\frac{2}{3}v(2c_0, \eta_0) \geq \theta \geq \frac{2}{7}v(2c_0, \eta_0) \tag{2}$$

Market shares of the two networks are then given by $\alpha_1 = \hat{x}$ and $\alpha_2 = 1 - \hat{x}$.

Setting the two-part tariff $(f_i, p_i, \widehat{p_i})$, the profit function for network i is equal to:

$$\begin{aligned} \pi_i\left(p_i, \hat{p}_i; p_{-i}, \hat{p}_j\right) &= \alpha_i \left\{\alpha_i q(p_i, \eta_i)(p_i - 2c_0) + (1-\alpha_i) q(\hat{p}_i, \eta_i)(\hat{p}_i - c_0 - a_j)\right. \\ &\quad \left. + (1-\alpha_i) q(\hat{p}_j, \eta_j)(a_i - c_0) + f_i\right\} \end{aligned}$$

where $q(p_i, \eta_i)$ and $q(\hat{p}_i, \eta_i)$ are respectively the number of on-net calls and off-net calls of network i.

¿From (**??**), we have $f_i = \alpha_i v(p_i, \eta_i) + (1-\alpha_i) v(\hat{p}_i, \eta_i) - w_i$, we can then rewrite profit of network i as:

$$\begin{aligned} \pi_i\left(\mathbf{p}, \mathbf{w}\right) &= \alpha_i \{\hat{x} q(p_i, \eta_i)(p_i - 2c_0) + (1-\alpha_i) q(\hat{p}_i, \eta_i)(\hat{p}_i - c_0 - a_j) + \\ &\quad + (1-\alpha_i) q(\hat{p}_j, \eta_j)(a_i - c_0) + \alpha_i v(p_i, \eta_i) + (1-\alpha_i) v(\hat{p}_i, \eta_i) - w_i\} \end{aligned} \tag{3}$$

In what follows we define two shortcut notations for indirect utility differences functions $V(y, z, i, j) \equiv v(y, \eta_i) - v(z, \eta_j)$ and revenues $R(y, z, t, i) \equiv (y - z) q(t, \eta_i)$.

Collusion. As is standard in the analysis of tacit collusion (Friedman (1971)), we consider an infinitely repeated tariff competition game. The punishment strategy for a given operator corresponds to a trigger strategy consisting of a reversion to static competitive equilibrium. We denote the individual profit gained from a punishment strategy (Nash reversion to competition) as π_i^*, and the individual collusion profit as π_i^C. Finally the individual profit gained from deviating from the collusive agreement is π_i^D. As is well know, the fully collusive outcome can then be sustained as a subgame-perfect equilibrium of the infinitely repeated game if the intertemporal discount factor, δ, is sufficiently large:

$$\delta_i \geq \widehat{\delta} = \max\{\widehat{\delta}_1, \widehat{\delta}_2\} \tag{4}$$

where $\widehat{\delta}$ denotes the critical discount factor and $\widehat{\delta}_i = \frac{\pi_i^D - \pi_i^C}{\pi_i^D - \pi_i^*}$ represents the critical discount factor for network i.

In the remainder of the paper, we will study levels and variations of this critical discount factor $\widehat{\delta}$ with respect to both reciprocity in the access charge regulation and potential asymmetry of networks. This will help us to asses how incentives to collude are driven by those features in this industry. Indeed, when for any reason, the critical discount factor decreases, firms are able to collude for a larger range of individual discount factors and conversely. As a result, any factor that pushes down that critical discount factor shall be considered has a factor that facilitates collusion in the industry. If it pushes up, the factor hinders collusion. Our aim is to indentify how asymmetric access charge regulation is such a facilitating or hindering factor when networks become asymmetric. To perform this analysis, we will now look at the equilibrium outcomes for each operator corresponding to the three different market configurations (competition, collusion and deviation).

3 Equilibrium outcomes

In this section, we determine the equilibrium outcomes for each market configuration (competition, collusion and deviation). Let's start with the competitive outcomes.

The competitive outcomes. This situation is the one studied by Laffont, Rey & Tirole (1998b). The equilibrium fixed fee and price vector components of network i satisfy: $(p_i^*, \hat{p}_i^*, f_i) = \arg\max_{p_i, \hat{p}_i, f_i} \pi_i\left(p_i, \hat{p}_i; p_j, \hat{p}_j\right)$. The result is then stated in the following Lemma.

Lemma 1 (LRT 1998a.). *The equilibrium unit prices and the fixed fee of network i in the competitive setting are:*

(i) $p_i^* = 2c_0$ *and* $\hat{p}_i^* = c_0 + a_j$

(ii) $f_i = \dfrac{\pi_i^*}{\alpha_i^*} - (1 - \alpha_i^*) R\left(a_i, c_0, \hat{p}_j^*, -i\right)$

Lemma 1 states the standard results for the competitive equilibrium prices. Equilibrium unit prices are equal to their respective marginal costs. Hence, on-net prices are set at the total marginal cost of an on-net call ($2c_0$) and off-net prices are set to their marginal cost including the unit termination fee of the competing network ($c_0 + a_j$), the total "perceived marginal cost". Equilibrium fixed fees are then used by networks to extract surplus from consumers.

It follows that equilibrium market shares as determined by the marginal consumer are

$$\alpha_1^* = \frac{\theta + V\left(\widehat{p}_1^*, p^*, 1, 2\right) - f_1 + f_2}{2\theta + V\left(\widehat{p}_1^*, p^*, 1, 1\right) + V\left(\widehat{p}_2^*, p^*, 2, 2\right)} \quad \text{and } \alpha_2^* = 1 - \alpha_1^*$$

Using (3), we obtain the competitive equilibrium profit of network i:

$$\pi_i^*\left(a_i, a_j, \eta_i, \eta_j\right) = \frac{\left(2\theta + \Sigma_{k=1}^{k=2} V\left(\widehat{p}_i^*, p^*, k, k\right) + R\left(a_i, c_0, \hat{p}_j^*, -i\right)\right)}{\left(6\theta + 2\Sigma_{k=1}^{k=2} R\left(a_{-k}, c_0, \hat{p}_k^*, k\right) + 3\Sigma_{k=1}^{k=2} V\left(\widehat{p}_k^*, p^*, k, k\right)\right)^2} \times$$
$$\times \left(3\theta + 2V\left(\widehat{p}_i^*, p^*, i, -i\right) + V\left(\widehat{p}_{-i}^*, p^*, -i, i\right) + \Sigma_{k=1}^{k=2} R\left(a_{-k}, c_0, \hat{p}_k^*, k\right)\right)^2$$

We highlight the fact that these equilibrium profits are functions of termination charges (a_1, a_2) and elasticity parameters (η_1, η_2). Note that when termination charges are cost based and symmetry holds, profit of network i is simply equal to $\pi_i^*\left(c_0, c_0, \eta_0, \eta_0\right) = \dfrac{\theta}{2}$.

The collusive outcomes. In order to determine the fully collusive outcome, we assume that the price vector maximizes joint profit subject to a participation constraint for all consumers. Then, collusive unit prices and the fixed fee result from $\max_{\mathbf{p},\mathbf{f}} \pi_1\left(\mathbf{p}, \mathbf{w}\right) + \pi_2\left(\mathbf{p}, \mathbf{w}\right)$ s.t. $U_k(x) \geq 0$. We, therefore, have the following result:

Lemma 2. *The equilibrium unit prices and the fixed fee of network i in the collusive setting are:*

(i) $p_i^C = \hat{p}_i^C = 2c_0$, *for* $i = 1, 2$

(ii) $f_i^C = \frac{1}{4}\left(3v(2c_0, \eta_i) + v(2c_0, \eta_j)\right) - \frac{1}{2}\theta$

Notice that this collusive equilibrium corresponds to the multiproduct monopolistic outcome when charging a two-part tariff. All collusive marginal prices are set to marginal cost in order to enhance network productive efficiency and the fixed fees are used to capture almost the entire consumer's surplus (and the entire one of the indifferent consumer).

Using (3) and substituting equilibrium collusive prices, we obtain the equilibrium profit of network i in the collusive setting:

$$\pi_i^C(a_i, a_{-i}, \eta_i, \eta_{-i}) = \frac{\left(2\theta + V\left(p^C, p^C, i, -i\right)\right)\left(2\theta + V\left(p^C, p^C, -i, i\right)\right)}{16\theta^2} \times$$
$$\times \left(R\left(a_i, c_0, p^C, -i\right) - R\left(a_{-i}, c_0, p^C, i\right) + \frac{\theta\left(3v\left(p^C, \eta_i\right) + v\left(p^C, \eta_{-i}\right) - 2\theta\right)}{\left(2\theta + V\left(p^C, p^C, -i, i\right)\right)} \right)$$

Note that when termination fees are cost based and symmetry holds, profits are simply $\pi_i^C(c_0, c_0, \eta_0, \eta_0) = (2v(2c_0, \eta_0) - \theta)/4$, which is positive if (2) holds.

The deviation outcomes. We assume w.l.o.g. that it is network i that deviates from the collusive outcome. Then, the deviation unit prices and the fixed fee is derived from $\max_{p_i, \hat{p}_i, f_i} \pi_i\left(p_i, \hat{p}_i, p_{-i}^C, \hat{p}_{-i}^C, f_i, f_{-i}^C\right)$ s.t. $U_k(x) \geq 0$. We then find the result:

Lemma 3. *The equilibrium unit prices and the fixed fee in the deviation setting are:*

(i) $p_i^* = 2c_0$ *and* $\hat{p}_i^* = c_0 + a_j$

(ii) $f_i^D = \dfrac{\pi_i^D}{\alpha_i^D} - (1 - \alpha_i^D) R(a_i, c_0, p^*, -i)$

Note that network i deviates from the collusive equilibrium using its fixed fee only while leaving unit on-net and off-net prices unchanged. Doing so, network i can attract more consumers and increase its overall profit.

Using (3), we deduce the equilibrium deviation profit of network i:

$$\pi_i^D(a_i, a_{-i}, \eta_i, \eta_{-i}) = \frac{1}{64} \frac{\left(4\,v\left(\hat{p}_i^*, \eta_i\right) + 4R\left(a_i, c_0, \hat{p}_{-i}^*, -i\right) + V(p^*, p^*, i, -i) + 2\,\theta\right)^2}{R\left(a_i, c_0, \hat{p}_{-i}^*, -i\right) + V(\hat{p}_i^*, p^*, i, i) + 2\,\theta}$$

Again, if reciprocal termination fees apply deviation profits are equal for the two operators. For both of them, cost-based termination fees and symmetry give profit: $\pi_i^D(c_0, c_0, \eta_0, \eta_0) = (2v(2c_0, \eta_0) + \theta)^2/32\theta$.

A thorny issue when looking at deviation outcomes is that monopolization can occur ex-post with the deviating firm remaining the only firm in the market. To avoid this we restrict our model to market conditions that preserve a duopolistic structure when firms deviate. With condition (2), network i's market share, α_i^D, always belongs to the interval $[0, 1]$ when $(a_i, a_{-i}, \eta_i, \eta_{-i}) = (c_0, c_0, \eta_0, \eta_0)$, for all i.

We are now in a position to construct and study the critical discount factor as defined in (4). However, due to the tedious expressions for profits we will not provide a complete exposure and characterization of this threshold. Note that the individual thresholds $\widehat{\delta}_i(a_i, a_{-i}, \eta_i, \eta_{-i})$ defined in (4) are implicit functions of $(a_1, a_2, \eta_1, \eta_2)$ as are profits.

Equipped with this framework we now proceed to the analysis of the sustainability of price collusion. We focus on the the effect of the asymmetry parameter influencing demand elasticity or network size, on the incentives for the operators to collude. In particular we examine how asymmetric termination fee regulation may affect the sustainability of collusion. However a complete analysis for all values of access charges (a_1, a_2) and asymmetry parameters (η_1, η_2) involves strong non-linearities that make the analysis very tedious. Hence, as it is standard in the literature, we will analyze asymmetric regulation locally around cost-based termination fees. That is, we will study in the following how a departure from cost-based regulation may affect the sustainability of collusion depending on the potential asymmetry of networks and regulation allowing for reciprocal or asymmetric termination fees. To better isolate the pure effect of network asymmetry we first consider that networks are symmetric, so that $\eta_1 = \eta_2 = \eta_0$, and study the impact of different regulatory termination fee regimes (reciprocal *v.s.* asymmetric regulation) on collusion.

4 Symmetric networks

This section analyzes the effects of asymmetric regulation[3], $a_1 \geq a_2 = c_0$, in the case where networks are symmetric, $\eta_1 = \eta_2 = \eta_0$. We first examine the reciprocal regulation regime and then asymmetric regulation.

4.1 Reciprocal regulation

Considering reciprocal regulation, $a_1 = a_2 = a$, with symmetric networks and a cost-based termination fee, $a = c_0$, it can be shown from (4) that the critical discount factor becomes:

$$\hat{\delta}(c_0, c_0, \eta_0, \eta_0) = \frac{2v(2c_0, \eta_0) - 3\theta}{2v(2c_0, \eta_0) + 5\theta}$$

Note that this corresponds to the long run situation in which the initial advantages of the incumbent (that may result from brand recognition or switching costs) are overcome and the networks' termination fees are regulated to follow for example the "glide path" to cost required by by the European Commission.

[3]Since networks are assumed to be symmetric, we could either consider the case $a_1 \geq a_2 = c_0$ or $a_2 \geq a_1 = c_0$.

The critical discount factor $\widehat{\delta}$ is then decreasing with the transportation cost θ, which plays the role of a network differentiation parameter. If θ is larger, goods become less substitutable for consumers, i.e. product differentiation is higher, which implies that it is easier to sustain collusion. Why is this the case? Omitting arguments, the competitive profit is

$$\pi_i^* = \frac{\theta}{2}$$

and, as usual in the Hotelling model, strongly increasing in θ. Note also that for $\theta > \frac{2}{3}v(2c_0, \eta)$, there is no incentive to deviate as

$$\pi_i^D = \frac{(2v(2c_0, \eta_0) + \theta)^2}{32\theta} < \pi_i^* = \frac{\theta}{2}.$$

Hence, we need (2) to hold[4]. The collusive profit is

$$\pi_i^C = \frac{2v\left(2c_0, \eta_0\right) - \theta}{4}$$

Note that the collusive profit is decreasing in θ, i.e. a higher degree of product differentiation reduces the total profit of an already colluding cartel, whereas the effect on the deviation profit is ambiguous. However, the differences in the numerator, $\pi_i^D - \pi_i^C$, and the denominator $\pi_i^D - \pi_i^*$ are decreasing in θ and, from the overall result, we know that the effect of the numerator dominates, making deviation from the collusive agreement less attractive.

Considering now reciprocal regulation, $a_1 = a_2 = a$. We have the following result:

Proposition 1. *In a symmetric network setting and with cost-based regulation, the critical discount factor $\widehat{\delta}$ is decreasing in the reciprocal access charge.*

We find that increasing a reciprocal termination fee locally around cost facilitates collusion. Conversely reducing reciprocal termination fee to cost as under the European glide path will make collusion harder to sustain. This first result underlines the collusive effect of reciprocal regulation in an infinitely repeated tariff competition game and confirms results of the standard literature on competition between interconnected networks stated by LRT (1998a) and Armstrong (1998). Indeed, when the access charge increases from its cost-base, off-net prices reach a higher level for both operators due to reciprocal access charges**.** Then operators compete through fixed fees which reduces their competitive profits. As a consequence they have a higher incentive to collude. This proposition shows that following the so-called European glide path gives a double-benifit for the society when network are (or have become) symmetric.

[4]For $\theta = \frac{2}{3}v(2c_0, \eta_0)$ we have $\pi_i^D = \frac{1}{3}v(2c_0, \eta_0)$ and $\pi_i^C = \frac{1}{3}v(2c_0, \eta_0)$ but also $\pi_i^* = \frac{1}{3}v(2c_0, \eta_0)$ then $\delta_i^* = 0$ and one can sustain collusion for any discount factor.

4.2 Asymmetric regulation

We now consider asymmetric regulation, i.e. non-reciprocal termination fees, $a_1 \geq c_0$ and $a_2 = c_0$. Operators then do not have the same incentives to collude and their critical discount factors take different values, $\widehat{\delta}_1 \neq \widehat{\delta}_2$, even though networks are fully symmetric. Of course such termination fees will have an impact on the incentive to collude for both networks and thus on the critical discount factor $\hat{\delta}$. Consider $a_2 = c_0$, the following proposition states the result for a slight deviation of network 1's termination fee from its cost-based level. Let's fist define $\widetilde{\theta} \equiv \frac{6}{13}v(2c_0, \eta_0)$. Then

Proposition 2. *With symmetric networks, assuming cost-based termination fees are regulated asymmetically, there exists a threshold* $\widetilde{\theta}$ *such that* $\frac{\partial\hat{\delta}(c_0,c_0,\eta_0,\eta_0)}{\partial a_1} \leq 0$ *if* $\theta \geq \widetilde{\theta}$ *and* $\frac{\partial\hat{\delta}(c_0,c_0,\eta_0,\eta_0)}{\partial a_1} > 0$ *otherwise.*

We find that when networks are symmetric, asymmetric regulation will facilitate collusion whenever the product differentiation is sufficiently high. Conversely, in that case, reducing asymmetric regulation toward a glide path regime will make collusion harder to sustain. However, this is no longer the case when product differentiation is low. It appears that if θ is low there may be a different impact of on-net and off-net access margins with respect to price collusion: mainly they facilitate collusion but they can hinder it if θ is high.

5 Asymmetric networks

We consider now that networks are asymmetric so that the asymmetry parameter is not the same for both networks. Let's suppose that network 1 has the higher value for the asymmetry parameter and network 2 still has the lower value η_0, so that $\eta_1 > \eta_2 = \eta_0$. Hence, different cases may arise whether network 1 benefits from asymmetric regulation or not. As in the previous section, we assume that asymmetric regulation benefits network 1, so $a_1 > a_2 = c_0$. Following Assumption 2, illustrated by *Examples* 1 and 2, the asymmetry between networks can represent two kind of situations. First, network asymmetry can fall fully on the demand elasticity (Assumption *A*.1). In this case, asymmetric regulation benefits the network with the higher elastricity. It has often been considered that new entrants in the mobile market face a higher elasticity than the incumbent because of switching costs or first mover advantage. Then, asymmetric regulation can be considered as a way to reduce the competitive disadvantage of the high elasticity network (i.e. new entrant) by offering the possibility to charge a higher termination fee than the incumbent. This has been allowed for in the European regulation of mobile termination rates. Network asymmetry can also fall on the demand or network size (Assumption $A.2$). This second case then represents the situation in which asymmetric regulation benefits the operator with the larger demand or network. Such kind of asymmetric regulation never happened in the termination fee regulation policy. This is probably because asymmetric regulation has always been implemented to limit the advantage of the incumbent and favour competition of new entrants. However, asymmetric regulation will have an effect on the sustainability of collusion when favoring the incumbent. That is, asymmetric regulation can reduce operators' incentives to stick to a collusive agreement. In the following, we investigate how the critical discount factor is affected by both networks asymmetry and different regulatory regimes.

5.1 Reciprocal regulation

Consider first that reciprocal regulation applies. Termination fees are then cost-based and $a_1 = a_2 = c_0$, the critical discount factors for network i is given by:

$$\hat{\delta}_i\left(c_0, c_0, \eta_i, \eta_{-i}\right) = \frac{9\left(v(2c_0, \eta_i) + 3v(2c_0, \eta_{-i}) - 6\theta\right)^2}{\left(23v(2c_0, \eta_i) - 11v(2c_0, \eta_{-i}) + 30\theta\right)\left(7v(2c_0, \eta_i) + 5v(2c_0, \eta_{-i}) - 18\theta\right)} \tag{5}$$

The following result compares the incentive for collusion of both networks and states the critical discount factor $\widehat{\delta}$:

Lemma 4. *Assuming a small asymmetry between networks (η_1 is in a right neighbourhood of $\eta_2 = \eta_0$):*

(i) if A.1 holds, then $\hat{\delta}_1 > \hat{\delta}_2$

(ii) if A.2 holds, then $\hat{\delta}_1 < \hat{\delta}_2$

When networks are asymmetric and the reciprocal termination fee is cost-based, the critical discount factor is the one corresponding to the operator that, because of a (perceived) differentiation in networks or the installed user base, is structurally able to lower the consumers' surplus at each price. The finding implies that a more advantaged firm is more likely to break a collusive agreement. This is in line with the common precept that collusion is easier to sustain among equals.

Let's now assume reciprocal termination fees, $a_1 = a_2 = a \geq c_0$ and assume a small termination fee mark-up. The following result shows what can be the interplay between network asymmetries and reciprocal termination fees.

Proposition 3. *(i) Firstly, when A.1. holds, there exists a level $\underline{v}$ of $v_2(p, \eta)$ and two values $\theta_1 = \underline{x}_1 v(2c_0, \eta_0)$ and $\theta_2 = \underline{x}_2 v(2c_0, \eta_0)$ where $\underline{x}_2 > \underline{x}_1 > 6/13$ such that:*

(i.a) if $\underline{v} < v_2(p,\eta) < 0$ *and* $\theta \in [\theta_1, \theta_2]$ *then*

$$\frac{\partial^2 \hat{\delta}_1 (c_0, c_0, \eta_0, \eta_0)}{\partial a \partial \eta_1} < 0$$

(i.b) if $\underline{v} < v_2(p,\eta) < 0$ *and* $\theta \notin [\theta_1, \theta_2]$ *or if* $v_2(p,\eta) \leq \underline{v}$ *for all* θ *then*

$$\frac{\partial^2 \hat{\delta}_1 (c_0, c_0, \eta_0, \eta_0)}{\partial a \partial \eta_1} > 0.$$

(ii) Secondly when A.2. holds, then unambiguously

$$\frac{\partial^2 \hat{\delta}_2 (c_0, c_0, \eta_0, \eta_0)}{\partial a \partial \eta_1} < 0$$

As shown in Proposition 1 when networks are homogeneous, reciprocal access charges over cost yield a *facilitating effect* on collusion in the industry. Proposition 3 shows that this facilitating effect is not always enhanced further by network asymmetries. This depend on both the level of product differentiation θ and the impact of network asymmetries on the surplus $v(p,\eta)$. It is worth pointing out that, depending on those fundamentals, the critical discount factor can be reduced when the reciprocal access charge is slightly raised above cost: More asymmetries are not hindering collusion systematically, this will be the case when asymmetries deteriorate surpluses strongly ($v_2(p,\eta) \leq \underline{v} < 0$).

5.2 Asymmetric regulation

Assuming non-reciprocal charges $a_1 \geq c_0$ and $a_2 = c_0$ from Proposition 2 we know the effects of access margins on collusion (around cost-based pricing) and from Proposition **??** we have results on the effects of network asymmetries. Using these results we now investigate how this result is affected by slight demand asymmetries.

Proposition 4. *Firstly, A.1 holds then there exists a level of* θ *i.e.* $\hat{\theta} < \frac{2}{7} v(2c_0, \eta_0)$ *such that*

$$\frac{\partial^2 \hat{\delta}_1 (c_0, c_0, \eta_0, \eta_0)}{\partial a_1 \partial \eta_1} \lesseqgtr 0 \text{ if } \theta \gtreqless \hat{\theta}$$

Secondly, when A.2 holds, then unambiguously

$$\frac{\partial^2 \hat{\delta}_2 (c_0, c_0, \eta_0, \eta_0)}{\partial a_1 \partial \eta_1} < 0$$

If *A.1.* holds (*i.e.* $v_2(p,\eta) < 0$) then a slight network asymmetry in the sense that $\eta_1 > \eta_0$ strengthens the facilitating effect of a positive off-net access margin on collusion if $\theta \geq \hat{\theta}$ but weakens this facilitating effect otherwise. If *A.2.* holds (*i.e.* $v_2(p, \eta) > 0$) then a similar slight network asymmetry always strengthens the facilitating effect of positive off-net access margins on collusion but weakens the *hindering effect* of positive off-net margins otherwise. Hence in both cases if product differentiation is high enough more asymmetries are not hindering collusion.

6 Conclusion

For a differentiated Bertrand duopoly setting, Baranes & Poudou (2009) show that cost symmetry may hinder collusion so that the common precept that it is easier to collude amongst equals does not always hold. In this case we look at a differentiated Hotelling duopoly model of the kind used by LRT (1998a,b) for the telecommunications industry with a potential asymmetry from differences in demand elasticities that may result from differences in firm histories.

We find that with *homogenous* networks, i.e. what will be the *long-run competitive outcome* in this industry following the technological breakthroughs that enabled liberalization and competition since the beginning of this century, under a cost based access charge regime, a larger reciprocal on-net off-net margin will actually improve the possibilities for collusion. We also found that reducing reciprocal access charges to true cost as aimed at by the European "glide path" envisaged by the Commission (and Ofcom in the UK, see Ofcom (2010)) will make collusion harder to sustain for homogenous networks.

In a competitive setting with *heterogenous* networks, i.e. what can be seen as the *medium term outcome* where competition is fostering and regulation can therefore be slowly fading out, a higher degree of differentiation in demand elasticities actually improves firms' profits and it is the firm facing the larger demand elasticity (usually the incumbent) that is more likely to have a level of impatience that leads to the breach of a collusive agreement. This has implications for policy for the medium term as measures aimed at equalizing (consumer's perception of the) networks may actually improve their possibilities to collude. The finding is thus in line with the common precept that it is easier to sustain collusion amongst equals and should keep regulators on their toes.

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Appendix

• **Proof of Lemma 1.** Given in LRT (1998a).. ■

• **Proof of Lemma 2.** Using expression (3), one can form the joint profit $\pi_1(\mathbf{p},\mathbf{w})+\pi_2(\mathbf{p},\mathbf{w})$ and shows ii is independent of (a_1,a_2), so the relevant first order conditions

$$\frac{\partial(\pi_1(\mathbf{p},\mathbf{w})+\pi_2(\mathbf{p},\mathbf{w}))}{\partial p_i}=0,\frac{\partial(\pi_1(\mathbf{p},\mathbf{w})+\pi_2(\mathbf{p},\mathbf{w}))}{\partial \hat{p}_i}=0$$

$\forall i\in\{1,2\}$ imply $p_1^C=\hat{p}_1^C=2c_0$. Then from since $\hat{x}=(\theta+w_1-w_2)/2\theta$, we have that

$$\hat{x}=\alpha=\frac{1}{2}+\frac{w_1-w_2}{2\theta}$$

and with (1) using $\mathbf{p}^C$ we can calculate

$$\hat{\alpha}^C = \frac{1}{2} + \frac{1}{2\theta}(v(2c_0, \eta_1) - f_1 - v(2c_0, \eta_2) + f_2)$$

Setting the utility of the marginal consumer to zero

$$\hat{U} = \hat{\alpha}^C v(2c_0, \eta_1) + (1 - \hat{\alpha}^C)v(2c_0, \eta_2) - f_1 = 0$$

one can determine the collusive fixed charge as

$$f_1^C = v(2c_0, \eta_1) + v(2c_0, \eta_2) - f_2 - \theta$$

Putting this into $\Pi = \pi_1(\mathbf{2c}_0, \mathbf{w}) + \pi_2(\mathbf{2c}_0, \mathbf{w})$ and maximizing it w.r.t. f_2, s.t. $U_2(\alpha_1^C) \geq 0$ yields the result

$$\mathbf{f}^C = \begin{pmatrix} f_1^C \\ f_2^C \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 3v(2c_0, \eta_1) + v(2c_0, \eta_2) - 2\theta \\ 3v(2c_0, \eta_2) + v(2c_0, \eta_1) - 2\theta \end{pmatrix}. \blacksquare$$

• **Proof of Lemma 3**. W.l.o.g. assume that $i = 1$. A similar proof holds if $i = 2$. From (3), the relevant first order conditions

$$\frac{\partial(\pi_1(p_1, \hat{p}_1, p_2^C, \hat{p}_2^C, f_1, f_2^C))}{\partial p_1} = 0 \text{ and } \frac{\partial(\pi_1(p_1, \hat{p}_1, p_2^C, \hat{p}_2^C, f_1, f_2^C))}{\partial \hat{p}_1} = 0$$

imply

$$(p_1^D, \hat{p}_1^D) = (p_1^*, \hat{p}_1^*) = (2c_0, c_0 + a_2)$$

i.e. optimal deviation yields "perceived marginal cost" pricing just as in monopoly. The deviant profit given $(p_1^D, \hat{p}_1^D)$ is

$$\pi_1^D = \hat{\alpha}^D\left((1 - \hat{\alpha}^D)\hat{q}_2(a_1 - c_0) + f_1^D\right)$$

and thus

$$f_1^D = \frac{\pi_1^D}{\alpha_1^D} - (1 - \alpha_1^D)q(\hat{p}_2^C, \eta_2)(a_1 - c_0).$$

with $\hat{p}_2^C = 2c_0 = p^*$. Moreover as $\hat{x} = \alpha$, with (1) and using $(p_1^D, \hat{p}_1^D)$, we can calculate

$$\hat{\alpha}^D = \frac{\theta - f_1 + f_2 - v(2c_0, \eta_2) + v(c_0 + a_2, \eta_1)}{2\theta - v(2c_0, \eta_1) + v(c_0 + a_2, \eta_1)}$$

Setting the utility of the marginal consumer to zero

$$\hat{U} = \hat{\alpha}^D v(2c_0, \eta_1) + (1 - \hat{\alpha}^D)v(c_0 + a_2, \eta_2) - f_1 - \theta\hat{\alpha}^D = 0$$

which can be solved for

$$f_2 = \frac{\theta f_1 + (\theta - v(2c_0, \eta_1) - v(2c_0, \eta_2)\theta - v(c_0 + a_2, \eta_1)v(2c_0, \eta_2) + v(2c_0, \eta_1)v(2c_0, \eta_2)}{v(2c_0, \eta_1) - v(c_0 + a_2, \eta_1) - \theta}$$

Plugging into the deviant profit

$$\pi_1^D = \hat{\alpha}_1^D \left((1-\hat{\alpha}_1^D)\hat{q}_2(a_1-c_0) + f_1^D\right)$$

and maximizing over f_1 one finds optimal deviation profit as

$$\pi_1^D = \frac{1}{64}\frac{\left(4\,v\left(\hat{p}_1^*,\eta_1\right) + v(p^*,\eta_1) - v(p^*,\eta_2) + 4\,(a_1-c_0)q\left(\hat{p}_2^*,\eta_2\right) + 2\,\theta\right)^2}{v\left(\hat{p}_1^*,\eta_1\right) - v(p^*,\eta_1) + (a_1-c_0)q\left(\hat{p}_2^*,\eta_2\right) + 2\,\theta}$$

Notice that when firms are homogeneous and access prices cost-based

$$\hat{\alpha}_1^D = \frac{2v(2c_0,\eta_0)+\theta}{8} = 1-\hat{\alpha}_2^D$$

We can check that $\hat{\alpha}_i^D \in [0,1]$ if (2) holds.

• **Proposition 1:** Using $\hat{\delta}_i = (\pi_i^D - \pi_i^C)/(\pi_i^D - \pi_i^*)$ with the profit terms for homogenous firms and reciprocal non-cost based access charge we take the derivative with respect to a and replace access charge with the true cost term c_0 to find:

$$\frac{\partial\hat{\delta}\left(c_0,c_0,\eta_0,\eta_0\right)}{\partial a} = -8\theta\frac{q(2c_0,\eta_0)}{(2v(2c_0,\eta_0)+5\theta)^2} < 0.$$

■

• **Proof of Proposition 2.** Denote the difference between both individual critical discount factors as $\Delta\left(a_1,a_2,\eta_1,\eta_2\right) \equiv \hat{\delta}_1\left(a_1,a_2,\eta_1,\eta_2\right) - \hat{\delta}_2\left(a_2,a_1,\eta_2,\eta_1\right)$. Around cost based access pricing for a_2, the variation of difference between critical discount factor $\hat{\delta}_1 - \hat{\delta}_2$ w.r.t. a_1 evaluated for $a_1 = c_0$ is written:

$$\lim_{a_1\to c_0}\frac{\partial\Delta\left(a_1,c_0,\eta_0,\eta_0\right)}{\partial a_1} = -\frac{16}{3}\frac{q(2c_0,\eta_0)(3v(2c_0,\eta_0)-5\theta)}{(2v(2c_0,\eta_0)+5\theta)^2}.$$

It is positive if $\theta \geq \frac{3}{5}v(2c_0,\eta_0)$ but negative if $\theta < \frac{3}{5}v(2c_0,\eta_0)$. Hence if $\theta \geq \frac{3}{5}v(2c_0,\eta_0)$, δ_1^* is the relevant critical discount factor as $\hat{\delta}_1 = \max\{\hat{\delta}_1,\hat{\delta}_2\}$ and taking the derivative with respect to a_1 for $a_1 = c_0$ leads to

$$\frac{\partial\hat{\delta}_1\left(c_0,c_0,\eta_0,\eta_0\right)}{\partial a_1} = -\frac{4}{3}\frac{q(2c_0,\eta_0)(6v(2c_0,\eta_0)-7\theta)}{(2v(2c_0,\eta_0)+5\theta)^2} < 0 \text{ for all } \theta$$

For $\theta < \frac{3}{5}v(2c_0,\eta_0)$, the relevant critical discount factor is then δ_2^* and the derivative with respect to a_1 for $a_1 = c_0$ writes

$$\frac{\partial\hat{\delta}_2\left(c_0,c_0,\eta_0,\eta_0\right)}{\partial a_1} = \frac{4}{3}\frac{q(2c_0,\eta_0)(6v(2c_0,\eta_0)-13\theta)}{(2v(2c_0,\eta_0)+5\theta)^2} \lesseqgtr 0 \text{ if } \theta \gtreqless \tilde{\theta}.$$

where $\tilde{\theta} = \frac{6}{13}v(2c_0,\eta_0)$. ■

• **Proof of Lemma 4.** Let us consider that $(a_1, a_2, \eta_1, \eta_2) = (c_0, c_0, \eta_1, \eta_0)$ and from (5) in the text one can form

$$\hat{\delta}_1(c_0, c_0, \eta_1, \eta_0) = 9\frac{(3\nu^0 + \nu^1 - 6\,\theta)^2}{(7\nu^1 + 5\nu^0 - 18\,\theta)(23\nu^1 - 11\nu^0 + 30\theta)}$$

$$\hat{\delta}_2(c_0, c_0, \eta_0, \eta_1) = 9\frac{(3\nu^1 + \nu^0 - 6\,\theta)^2}{(7\nu^1 + 5\nu^0 - 18\,\theta)(23\nu^1 - 11\nu^0 + 30\theta)}$$

where $\nu^0 = v(2c_0, \eta_0)$ and $\nu^1 = v(2c_0, \eta_1)$. Thus we can derive $\Delta(c_0, c_0, \eta_1, \eta_0)$ and considering now a slight increase in η_1 above η_0 then we have

$$\frac{\partial\Delta(c_0, c_0, \eta_0, \eta_0)}{\partial\eta_1} = -\frac{16}{3}v_2(2c_0, \eta_0)\frac{3\nu^0 - \theta}{(2\nu^0 + 5\theta)^2} \quad \text{(A.1)}$$

as θ has been assumed to take values below $\frac{2}{3}\nu^0$, the sign of this derivative is exactly the opposite of the sign of $v_2(p, \eta)$. Hence it depends on assumptions *A.1* and *A.2*.
■

• **Proof of Proposition 3.** If *A.1* holds i.e. $v_2(p, \eta) < 0$ and $q_2(p, \eta) < 0$, the following second order cross partial derivative tells us how this reciprocal access pricing effect is modified by an increasing network asymmetry, that is

$$\frac{\partial^2\hat{\delta}_1(c_0, c_0, \eta_0, \eta_0)}{\partial a\partial\eta_1} = \frac{4}{3}\frac{(6\nu^0 - 13\theta)}{(2\nu^0 + 5\theta)^2}q_2 - \frac{8}{9}\frac{(8(\nu^0)^2 - 154\theta\nu^0 + 117\theta^2)\,q^0}{(2\nu^0 - 3\theta)(2\nu^0 + 5\theta)^3}v_2$$

with $\nu^0 = v(2c_0, \eta_0)$, $q^0 = q(2c_0, \eta_0)$, $v_2 = v_2(2c_0, \eta_0)$ and $q_2 = q_2(2c_0, \eta_0)$. Letting $\theta = xv^0$ for $x \leq \frac{2}{3}$, first one can easily see that $(8 - 154x + 117x^2)(v^0)^2 < 0$ if (2) holds i.e. if $x \in [\frac{2}{7}, \frac{2}{3}]$. Hence if $x \in [\frac{2}{7}, \frac{6}{13}]$, then unambiguously (omitting arguments) $\partial^2\delta_1^*/\partial a\partial\eta_1 < 0$. However, if $x > \frac{6}{13}$, we can define values $\underline{y}(x)$ of v_2 such that $\partial^2\delta_1^*/\partial a\partial\eta_1$ reaches zero for each $x \in]\frac{6}{13}, \frac{2}{3}]$. This leads to solve $\partial^2\delta_1^*/\partial a\partial\eta_1 = 0$ with respect to v_2, that is

$$\underline{y}(x) = \frac{3}{2}\frac{(6 - 13x)(2 - 3x)(2 + 5x)}{8 - 154x + 117x^2}\frac{\nu^0}{q^0}q_2 < 0 \text{ for all } x \in]\frac{6}{13}, \frac{2}{3}]$$

Moreover studying $\underline{y}(x)$ shows that, in the interval $]\frac{6}{13}, \frac{2}{3}]$, it reaches its minimum for $x = \underline{x}$ where $\underline{y}'(\underline{x}) = 0$ such as

$$\underline{x} \in \arg\left\{x \in]\frac{6}{13}, \frac{2}{3}]\,\middle|\, 3472 - 7888x + 29824x^2 - 60060x^3 + 22\,815x^4 = 0\right\}$$

so that $\underline{x} \simeq 0.5568$ and $\underline{y}(\underline{x}) \simeq 0.107\frac{\nu^0}{q^0}q_2 < 0$. Therefore we can conclude that if $v_2 < \underline{y}(\underline{x})$ hence $\partial^2\delta_1^*/\partial a\partial\eta_1 < 0$ for all admissible x. But if $\underline{y}(\underline{x}) < v_2 < 0$, as $\partial^2\delta_1^*/\partial a\partial\eta_1$ increases with v_2, $\partial^2\delta_1^*/\partial a\partial\eta_1 > 0$ at $x = \underline{x}$. As $\underline{y}(x)$ is strictly convex in x, for each v_2, it exists two values of $\underline{x}_1$ and $\underline{x}_2$, such that $\underline{x}_2 < \underline{x} < \underline{x}_1 < \frac{6}{13}$, defined by $\underline{y}(\underline{x}_1) = \underline{y}(\underline{x}_2) = v_2$ and for which $\partial^2\delta_1^*/\partial a\partial\eta_1 > 0$ when $x \in [\underline{x}_1, \underline{x}_2]$.

Second assume that *A.2* holds i.e. $v_2(p,\eta) \geq 0$ and $q_2(p,\eta) \geq 0$, in same way as above, one can find that

$$\frac{\partial^2 \delta_2^* \left(c_0, c_0, \eta_0, \eta_0\right)}{\partial a \partial \eta_1} = -\frac{4}{3} \frac{\left(6\nu^0 - 7\theta\right)}{\left(2\nu^0 + 5\theta\right)^2} q_2 + \frac{8}{9} \frac{\left(8(\nu^0)^2 - 82\theta\nu^0 + 9\theta^2\right) q^0}{\left(2\nu^0 - 3\theta\right)\left(2\nu^0 + 5\theta\right)^3} v_2$$

Letting $\theta = xv^0$ for $x \leq \frac{2}{3}$, one can easily see that $\left(8x^2 - 82x + 9\right)\theta^2 < 0$ and $\left(6x - 7\right)\theta > 0$ if (2) holds. A a result, $\partial^2\delta_2^*/\partial a \partial \eta_1 < 0$ for all $x \leq \frac{2}{3}$. ■

• **Proof of Proposition 4.** From Lemma 4 we know that around cost-based access pricing for a slight asymmetry of network 1 such that $\eta_1 > \eta_2 = \eta_0$ the critical factor is δ_1^* if $v_2(p,\eta) < 0$ and δ_2^* if $v_2(p,\eta) > 0$. If *A.1* holds i.e. $v_2(p,\eta) < 0$, the following second order cross partial derivative tells us how this access pricing effect is modified by an increasing network asymmetry, that is

$$\frac{\partial^2 \delta_1^* \left(c_0, c_0, \eta_0, \eta_0\right)}{\partial \eta_1 \partial a_1} = -\frac{4}{9} \frac{\left(36\left(v^0\right)^3 - 188\theta\left(v^0\right)^2 + 637\theta^2\nu^0 - 718\,\theta^3\right) q^0}{\theta\left(2\nu^0 - 3\theta\right)\left(2\nu^0 + 5\theta\right)^3} v_2$$

where $\nu^0 = v(2c_0,\eta_0)$, $q^0 = q(2c_0,\eta_0)$, $v_2 = v_2(2c_0,\eta_0)$. Let $\theta = xv^0$ for $x \leq \frac{2}{3}$, thus we have $\left(36 - 188x + 637x^2 - 718x^3\right)\left(v^0\right)^3 \geq 0$ and so $\frac{\partial^2\delta_1^*(c_0,c_0,\eta_0,\eta_0)}{\partial\eta_1\partial a_1} > 0$ if $x \geq 0.586$ and negative otherwise. Hence in the first case of Proposition 2 we have shown that around symmetry for networks if $\theta > \frac{3}{5}v^0 > 0.586v^0$ then $\frac{\partial^2\delta_1^*(c_0,c_0,\eta_0,\eta_0)}{\partial\eta_1\partial a_1} < 0$ and by $\frac{\partial\delta_1^*(c_0,c_0,\eta_0,\eta_0)}{\partial a_1} < 0$. If *A.2* holds i.e. $v_2(p,\eta) \geq 0$ the corresponding derivative on the critical discount factor δ_2^* writes

$$\frac{\partial^2 \delta_2^* \left(c_0, c_0, \eta_0, \eta_0\right)}{\partial \eta_1 \partial a_1} = -\frac{4}{9} \frac{\left(36\left(v^0\right)^3 - 132\theta\left(v^0\right)^2 + 345\theta^2\nu^0 - 214\theta^3\right) q^0}{\theta\left(2\nu^0 - 3\theta\right)\left(2\nu^0 + 5\theta\right)^3} v_2$$

Using again the variable change $\theta = xv^0$ for $x \leq \frac{2}{3}$, we have $\left(36 - 132x + 345x^2 - 214^3\right)\left(v^0\right)^3 \geq 0$ for all x, and so $\frac{\partial^2\delta_2^*(c_0,c_0,\eta_0,\eta_0)}{\partial\eta_1\partial a_1} < 0$. So from Proposition 2 we know that $\frac{\partial\delta_2^*(c_0,c_0,\eta_0,\eta_0)}{\partial a_1} \lesseqgtr 0$ as $\theta \gtreqless \frac{6}{13}\nu^0$. Then for θ "large" we have $\frac{\partial\delta_2^*(c_0,c_0,\eta_0,\eta_0)}{\partial a_1} < 0$ and the critical discount factor is equally reduced further by (slight) network asymmetries. ■