Game of Platforms: Strategic Expansion in Two-Sided Markets*

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Abstract

Online platforms, such as Google, Facebook, or Amazon, are constantly expanding their activities, while increasing the overlap in their service offering. In this paper, we study the scope and overlap of online platforms' activities, when they are endogenously determined.

We model an expansion game between two online platforms offering two different services to users for free, while selling user clicks to advertisers. At the outset, each platform offers one service, and users may subscribe to one platform or both (multi-homing). In the second stage, each platform decides whether to expand by adding the service already offered by its rival. Platforms' expansion decisions affect users' mobility, and thus the partition of users in the market, which, in turn, affects platform prices and profits.

We analyze the equilibrium of the expansion game, demonstrating that, in equilibrium, platforms may decide not to expand, even though expansion is costless. Such strategic "no expansion" decisions are due to quantity and price effects of changes in user mobility, brought on by expansion. Both symmetric expansion and symmetric no-expansion equilibria may arise, as well as asymmetric expansion equilibria, even for initially symmetric platforms.

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1 Introduction

Online platforms, such as Google and Facebook are constantly expanding their activities. Google, since its founding, has added and continues to add many products and services to its core search engine, including email, an online office suite, a social networking service, cloud computing, and many more. Facebook, whose core activity is social networking, continues to augment its core service with related services, including email, instant messaging, and very recently - Facebook graph search. Evidently, in their expansion efforts, the two companies are venturing into each other's territories, increasing the overlap in their activities.

Both Facebook and Google's revenue comes predominantly from advertising, and this is also the case for many other online platforms. Advertising revenues are usually collected on a per-click basis, whereby advertisers pay the platform a price per each user click on their ads. Users often enjoy the services provided by the platforms for free, and are thus fully subsidized by the advertisers' side of the market.

This work studies the expansion behavior of such online platforms, within a game theoretic framework. We model a market with two platforms, through which *advertisers* may reach potential *buyers*, also referred to as *platform users*. Initially, the platforms provide two different types of services. The platforms provide their services free to users, generating revenues by selling user clicks to advertisers.[1] At the outset, each platform has an exogenously given set of users, some of which *multihome* by subscribing to both platforms. This initial partition of users may take different forms, with varying degrees of asymmetry. The initial user partition represents platforms' installed base, which is the result of past events outside the scope of the model (e.g. past interaction between the platforms, or different dates at which the platforms began operating in the market).

Platforms play an entry or *expansion* game, where each platform strategically chooses whether or not to add the service already offered by the rival platform. Such expansion is assumed to be costless in the model. The game proceeds in two stages. In the first stage, platforms make their expansion decisions, which determine the set of services offered in the market. The number of services offered also affects the level of user mobility, i.e., the fraction of users who may switch between service providers. Users observe platforms' service offering following expansion decisions, and the non-captive users choose which platform(s) to join, thus creating the final user partition in the market. In the second stage, platforms play a pricing game, in which they set prices per user click charged to advertisers in the market. Advertisers then observe platform prices and users' final partition, and choose where to place their ads,

[1] The advertisers' side of the market fully subsidizes the buyers' side. This price structure is a standard result for media and advertising markets, and follows from a high price sensitivity on the buyers' side, together with a strong positive network externality exerted by potential buyers on advertisers. We abstract away from assumptions regarding buyers' price sensitivity, and assume the above price structure, as justified by previous work (see Rochet and Tirole 2006, and Armstrong 2006 for the theoretical basis, and Kaiser and Wright 2006 for empirical validation).

which, in turn, determines platforms' profits.

We now specify our main assumptions regarding users, advertisers and platforms in the market. As noted, users are characterized by imperfect mobility, such that some, but not all, consider switching platforms when observing the service offering following expansion decisions. The remaining users are captive, and continue subscribing to their original platform, regardless of its expansion decision. We assume that non-captive users prefer subscribing to both service types over subscription to just one service. As a base case, we consider migrating users who become multihomers (i.e., subscribers of the two original services). This could result from a higher quality for platforms' core services.[2] The alternative case where non-captive users prefer an expanded platform to multihoming (due to low compatibility or platform related network effects) is discussed in section 5.

The level of mobility in the market is defined as the fraction of non-captive users, and is determined endogenously by platform expansion decisions. Specifically, mobility may either increase or decrease with the number of services available. We first analyze the expansion game for the case where user mobility increases with the introduction of new services. Mobility may increase in the number of services, if we consider users who are encouraged to try out different providers when facing more options in the market. The analysis is then conducted for the case where mobility decreases with platform expansion. This is possible if we consider a non-expanding platform as specializing in its core service. When this is the case, fewer services imply higher specialization, and therefore an increased tendency to multihome, i.e. mobility that decreases in the number of services offered.

Platforms sell user clicks to advertisers, where these clicks are either generated by the platforms' exclusive users or by multihomers who subscribe to each platform's original service. The expected number of clicks sold for each type of users (exclusive or multihomer) is the product of the number of platform subscribers and their click-through rate (henceforth, CTR). The CTR is the probability that a platform user clicks on an ad viewed on the platform. Expansion increases the CTR only for exclusive users, as exclusive users are exposed to ads on two services rather than one, following expansion. Expansion does not increase the CTR for multihomers, which remains at the baseline level - the one-service CTR. It is important to note that the two-service CTR (i.e. CTR for exclusive users following expansion) is higher, but less than double, the one-service CTR. Further note that each platform sets a single price per click, which is the same for exclusive and multihoming users.

Advertisers place ads on platforms, and pay platforms for each user click on their ads. Advertisers' willingness to pay, or value, per click is constant and normalized to one. Advertisers choose where to advertise, and may advertise on one of the platforms, on both, or not advertise at all. Advertisers' choice maximizes their total expected value from advertising. Total

[2] Alternatively, movers' choice of multihoming could be the result of a positive same-side network effect for services based on the initial partition, such that services with more subscribers at the outset are preferred. This network effect (trivially) implies that migrating users subscribe to the two original services, since newly added services had no subscribers in the initial partition.

expected value for advertisers is defined as the difference between the value of expected clicks on ads (i.e., per click value times expected number of clicks), and their cost, which equals price per click times the expected number of clicks. Recall that the price per click is the same for all clicks generated by the platform, whereas the CTR guaranteed by an expanded platform is not the same for exclusive and multihoming users.

Given a pair of expansion decisions, the resulting user partition, and advertisers' choice rule, platforms engage in a pricing game. The main feature of the resulting prices is that prices per click decrease with the size of the multihoming group. This is the case because, in the pricing game equilibrium, prices are set such that advertisers place ads on both platforms, but do not pay double for multihomers' clicks. Specifically, both platforms sell clicks by multihomers, and expected payment to each platform for these clicks is derived according to the one-service CTR. However, with advertising on both platforms, multihomers' expected number of clicks is derived according to the two-service CTR. Since the two-service CTR is less than double the one-service CTR, reaching multihomers using ads on both platforms creates some redundancy. Platforms' equilibrium per click prices are therefore constrained by the degree of user multihoming. The multihoming group thus determines platforms' market power in the model, such that a low degree of multihoming implies a high degree of market power, with high per-click prices, and a large degree of multihoming lowers platforms' market power and the prices per clicked charged.

Market equilibrium is users' final partition, advertisers' choice of where to advertise, and platforms' expansion and pricing strategies, such that: (1) Users' final partition is a function of the level of mobility, determined by platforms' expansion decisions, and users' choice rule; (2) Advertisers' platform choice is optimal, given platforms' expansion decisions, the resulting user partition, and platform pricing; (3) Platform pricing is Nash equilibrium, given their expansion decisions, and advertisers' choice rule; and (4) Platforms' expansion decisions are Nash equilibrium in the expansion game, where profits in each subgame are determined by (1)-(3).

The game is solved by backward induction. For each of four expansion subgames, the resulting user partition is derived and the platform pricing game is solved. Platform profits in each subgame then follow. This yields the payoff matrix for the platform expansion game, which is then solved to determine the expansion equilibrium. We consider both a simultaneous and a sequential move game. The simultaneous move game represents the case where platforms' development strategies are kept secret until the new service is launched, whereas the sequential version represents a market where platforms' development strategies are known. For the sequential move expansion game, we consider both the case where the larger platform is the first mover, and the case where the smaller platform is first to decide on its expansion strategy.

Platform expansion has two main effects in the model. The first is the *CTR effect.* Expansion increases the CTR for platforms' exclusive users, from the one-service to the two-service

CTR. The CTR effect is thus positive, driving towards expansion. The second effect created by expansion decisions is the *mobility effect*, as the level of mobility changes with the number of services available. The mobility effect implies that expansion decisions affect the final user partition, which in turn affects the pricing game and platforms' profits for each pair of expansion decisions. The mobility effect is both direct and indirect. The direct mobility effect is the quantity effect, i.e. the change in the number of exclusive and multihoming clicks sold by each platform, resulting directly from the change in the final user partition. Note that this effect differs in magnitude, and possibly in direction, for the large and small platforms. This is because the smaller platform enjoys a relatively larger benefit from its rival's loss of subscribers. The indirect mobility effect is the price effect created by changes in the degree of multihoming, where increased multihoming lowers platform prices. This price effect of mobility may thus counter the CTR effect, and positive quantity effects, leading to equilibria where one or both platforms choose no expansion.

Equilibrium expansion decisions are derived separately for the case where expansion increases mobility, and for the case where mobility is decreased by expansion. When expansion increases mobility, the price effect of expansion is negative, while the quantity effect depends on platforms relative size, and on the opponent's expansion decision. Expansion decisions, in this case, will depend on the CTR. For low CTRs, the CTR effect dominates and the equilibrium is symmetric expansion. As the CTR increases, and the CTR effect decreases in magnitude, the negative price effect along with asymmetric quantity effects will lead to asymmetric expansion equilibria for mid-level CTRs, and to symmetric no-expansion for high CTRs.

Alternatively, when expansion decreases buyer mobility, then it increases both the CTR and exclusivity in the market, i.e. the CTR and price effects operate in the same direction. Expansion decisions, in this case, will depend on the magnitude of change in mobility, and on its initial level. When the change in mobility is small the CTR and price effects dominate, and the equilibrium is symmetric expansion. When the change in mobility is large, expansion's effect of reduced migration by the opponent's subscribers becomes pronounced. This may result in an equilibrium with asymmetric expansion, or an equilibrium with symmetric no-expansion.

The paper is related to the burgeoning literature on two-sided markets. A large part of this literature has focused on questions regarding platforms' optimal price structure, price competition, and identifying which platform will "win the market" following price competition. Such work has been carried out in the context of different market environments, with varying assumptions regarding preferences of end users, their beliefs, and the information structure (examples include Caillaud and Jullien 2001 and 2003, Roche and Tirole 2003 and 2006, Armstrong 2006, Hagiu 2009, and Yehezkel and Halaburda 2011).

Fewer papers have examined platform strategies other than pricing. Strategies studied include tying (Amelio and Jullien 2012, Choi 2011), and compatibility (Casadesus-Masanell and Ruiz-Aliseda 2009). In the context of newspaper markets, several papers have studied newspapers' strategic choice of differentiation (Gabszewicz, Laussel and Sonnac 2001 and 2002, Behringer and Filistrucchi 2009).

This work focuses on online platforms' entry decisions, analyzing their strategic choice of services and the degree of overlap in the services offered. This research thus contributes to the literature by shedding light on an important facet of the strategic behavior of online platforms - the ongoing strategic expansion which is shaping online markets.

2 The model

We model a market with two platforms, offering online services to *buyers* or platform *users*, in order to attract *advertisers*. At the outset, each platform offers one type of service, different from that offered by its rival. The initial partition of buyers is exogenously given, i.e. each platform has a given group of users, some of which multihome by subscribing to both platforms. Platform revenues are generated from the advertisers' side of the market, as platforms charge advertisers a price per each user click on ads displayed by the platform.

The focus of the model is platforms' strategic expansion behavior. We analyze an entry or *expansion* game between the two platforms, where each platform may or may not expand by adding the service initially offered by the rival platform. Following platform expansion decisions, some, but not all, users choose which platform(s) to join, and the number of services available affects the level of user mobility. Platform expansion decisions thus determine the final buyer partition in the market. Platforms then set prices per user click charged to advertisers. Advertisers observe the partition of buyers, and platforms' prices, and choose on which platform(s) to place their ads.

2.1 Platforms - basic assumptions and notation

There are two platforms in the market and let $i \in \{1, 2\}$ denote the platform index. Platforms provide free services to their users, generating revenues by charging the advertiser side of the market a price $p_i \in [0, \infty)$ for each user click on an ad displayed by the platform. At the outset, each platform is the provider of one type of service, different from the type offered by the rival platform.

A strategy for platform i is a couple (e_i, p_i) where $e_i \in \left\{E, \bar{E}\right\}$ represents the platform's expansion decision that can be either "expansion" denoted E or "no expansion" denoted $\bar{E}$; and $p_i \in [0, \infty)$ is the price per user click charged to advertisers on the platform. We assume that each platform may expand only by adding the service already offered by its rival.

Platform profits are derived from user clicks sold to advertisers. We thus turn to introduce assumptions regarding buyers and advertisers in the market.

2.2 Buyers

There is a unit mass of buyers in the market. The partition of buyers at the outset is denoted $B = \{b_1, b_2, b_{12}\}$, where b_i is the group of platform i's exclusive subscribers and its mass, and b_{12} is the initial group of multihomers, and its mass (with some abuse of notation). Multihomers are buyers that subscribe to both platforms, using each platform's original or *core* service.

The initial partition of buyers is exogenously given, and is not the result of optimization, representing each platform's installed base. This implies that there may be asymmetry in platform size at the outset, where platform size is defined as the number of exclusive platform subscribers. WLOG let $b_1 \geq b_2$, and let $\delta \equiv b_1 - b_2 \geq 0$ denote the difference in platforms' initial size. We will focus our analysis on the effects of the degree of multihoming, and on the effects of the initial asymmetry in the market. We thus write group sizes as a function of b_{12} and δ:

$$b_1 = 0.5\,(1 - b_{12} + \delta) \tag{1}$$

$$b_2 = 0.5\,(1 - b_{12} - \delta) \tag{2}$$

Users' click-through rate. Click-through rate (henceforth, CTR) is the probability that a user clicks on an ad, and thus increases in the number of exposures to the ad. Ad exposure occurs via platforms' services. This implies that exclusive users are exposed to ads once when a platform has not expanded, and twice when it has (given that advertisers place ads on said platform). Multihomers are exposed to ads once if advertisers place ads on one of the platforms, and twice if advertisers opt to advertise on both platforms.

Let $\rho \in (0, 1)$ denote the one-service CTR, i.e. the probability a user clicks on an ad he sees on one service, and let $\tilde{\rho}$ denote the two-service CTR, i.e. the probability he clicks on an ad following exposure on two services. The two-service CTR is given by $\tilde{\rho} = 2\rho - \rho^2$, which is the probability that a user clicks at least once in two exposures. This definition of $\tilde{\rho}$ represents an underlying assumption that users do not click twice on the same ad; specifically, advertising on two services reduces the probability of a "no click" event, but the two-service CTR is not double the one-service CTR - $\rho < \tilde{\rho} < 2\rho$.

Imperfect mobility. We assume that the initial partition is "sticky", such that, for each pair of expansion decisions (e_1, e_2), only fraction $\beta^{e_1 e_2} \in (0, 1)$ of buyers may switch platforms or become multihomers, and the remaining $(1 - \beta^{e_1 e_2})$ buyers are captive users of the platform(s) they initially subscribed to. We refer to $\beta^{e_1 e_2}$ as the level of *mobility* in the market, and to the corresponding group of non-captive, migrating, users as *movers*. Note that we continue to assume that multihomers subscribe to the two original services, regardless of (e_1, e_2).

Movers' choice rule. The non-captive users are assumed to be homogeneous, and employ the same choice rule. We assume that movers prefer subscription to both service types over just one. This implies that movers choose either multihoming or an expanded platform (if one or both platforms expand). As a base case, we consider migrating users who choose to multihome. This could result from a higher quality for the core services, coupled with a high level of compatibility between the two platforms. In section 5, we discuss the alternative case where non-captive users prefer an expanded platform over multihoming, due to low compatibility or a strong platform-related network effect.

The level of mobility. The level of mobility in the market depends on platforms' expansion decisions, and specifically assume that mobility changes monotonically with the number of services offered. Denote by $\beta^n \in (0,1)$ the level of mobility for a market with n new services, where $n \in \{0, I, II\}$. This implies that $\beta^{\bar{E}\bar{E}} = \beta^0$, $\beta^{\bar{E}E} = \beta^{E\bar{E}} = \beta^I$, and $\beta^{EE} = \beta^{II}$. We consider the following two separate cases:

1. Mobility increases in the number of services offered: $\beta^0 < \beta^I < \beta^{II}$. This represents markets where users are more likely to sample and change service providers, when their choice set increases.

2. Mobility decreases in the number of services offered: $\beta^0 > \beta^I > \beta^{II}$. This assumption is well suited for our base case where non-captive users prefer to multihome, and no-expansion implies specialization in platforms' core activity. When this is the case, the tendency to multihome is higher when fewer new services are introduced, i.e. - mobility decreases in n.[3]

For both cases, we assume, for simplicity, that the change in mobility resulting from the introduction of one service is constant, such that:

$$\Delta\beta \equiv \beta^{II} - \beta^I = \beta^I - \beta^0 \tag{3}$$

We further assume that changes in mobility are bounded, and specifically that $\Delta\beta \in \left(-0.5\beta^0, 0.5\left(1-\beta^0\right)\right)$, to ensure that $\beta^2 \in (0,1)$.

Buyers' final partition. The final partition of buyers is denoted $\bar{B} = \{\bar{b}_1, \bar{b}_2, \bar{b}_{12}\}$, where $\bar{b}_i$ denotes both the group of platform i's subscribers and its mass, following platforms' expansion decisions and movers' choice rule; $\bar{b}_i$ is given by:

$$\bar{b}_i = \left(1 - \beta^{e_1 e_2}\right) b_i + \Delta b_i \tag{4}$$

Where Δb_i depends on movers' choice rule. $\bar{b}_{12}$ is similarly defined.

In the base case, with multihoming movers $\Delta b_i = 0$, and the mass of multihomers in the final partition is given by $\bar{b}_{12} = \left(1 - \beta^{e_1 e_2}\right) b_{12} + \beta^{e_1 e_2}$.

Summarizing, buyers' final partition $\bar{B}$ depends on platforms' entry decisions, and is derived from the initial partition, together with movers' choice rule. Buyers' final partition then affects advertisers' choice of platforms.

[3] Alternatively, one may think of users that become confused by the introduction of new services, such that they are less likely to sample and change providers when their choice set increases.

2.3 Advertisers

There is a unit mass of homogeneous advertisers in the market. Advertisers' strategy is a choice of platform or platforms on which to place ads, denoted $\alpha \in A \equiv \{\{1\},\{2\},\{1,2\},\emptyset\}$.

Advertisers' value for each user click on their ads is constant, and normalized to 1. The expected value of advertising on α, depends on buyers' final partition, the CTR for buyers reached through α, and the price per click charged. The expected value for an advertiser from choice α, given $(e_i, p_i)_{i=1,2}$, and $\bar{B}$, is denoted $V^\alpha \equiv V\left(\alpha \mid (e_i, p_i)_{i=1,2}, \bar{B}\right)$:

$$V^\alpha = \begin{cases} (1-p_i)\left[\rho_i \bar{b}_i + \rho \bar{b}_{12}\right] & \text{for } \alpha = \{i\} \\ \left[\rho_1 \bar{b}_1 + \rho_2 \bar{b}_2 + \tilde{\rho} \bar{b}_{12}\right] - p_1\left[\rho_1 \bar{b}_1 + \rho \bar{b}_{12}\right] - p_2\left[\rho_2 \bar{b}_2 + \rho \bar{b}_{12}\right] & \text{for } \alpha = \{1,2\} \\ 0 & \text{for } \alpha = \emptyset \end{cases} \tag{5}$$

Where ρ_i is the CTR for exclusive subscribers of platform i, and thus depends on its expansion decision:

$$\rho_i = \begin{cases} \rho & e_i = \bar{E} \\ \tilde{\rho} & e_i = E \end{cases} \tag{6}$$

Whenever advertising on platform i, advertisers pay the platform for the user clicks it provides. The expected number of clicks generated by advertising on platform i is $\left[\rho_i \bar{b}_i + \rho \bar{b}_{12}\right]$, where $\rho_i \bar{b}_i$ are expected clicks from i's exclusive subscribers, and $\rho \bar{b}_{12}$ are expected clicks of multihomers, who subscribe to one service offered by i. Therefore, advertising on platform i is always at an expected cost of $p_i\left[\rho_i \bar{b}_i + \rho \bar{b}_{12}\right]$.

The expected value of advertising on a single platform is the product of advertiser's per click value, and the expected number of clicks, minus the expected cost of advertising, $(1-p_i)\left[\rho_i \bar{b}_i + \rho \bar{b}_{12}\right]$. When advertising on both platforms, an advertiser reaches multihoming buyers through two services, and therefore the expected number of clicks generated by this group is $\tilde{\rho} \bar{b}_{12}$, while each platform charges $p_i \cdot \rho \bar{b}_{12}$ for multihomers' clicks. The expected value from the choice $\alpha = \{1,2\}$ is therefore smaller than the sum of values generated by advertising on each platform separately.

Given platforms' entry and pricing decisions, and the resulting buyer partition, an advertiser chooses where to advertise, α^*, so as to maximize his expected value from advertising:

$$\alpha^* = \arg\max_{\alpha \in A} \left\{V\left(\alpha \mid (e_i, p_i)_{i=1,2}, \bar{B}\right)\right\} \tag{7}$$

2.4 Platforms - profits and optimization

Each pair of platform expansion decisions (e_1, e_2) defines an expansion subgame, and determines $\bar{B}$ in the subgame. Platform i's expected profit in an expansion subgame (e_1, e_2), for price p_i, given $\bar{B}$ and the rival's price p_j, are denoted $\pi_i\left(p_i|p_j, e_1, e_2, \bar{B}\right)$, and given by:

$$\pi_i\left(p_i|p_j, e_1, e_2, \bar{B}\right) = \begin{cases} p_i \cdot \left[\rho_i \bar{b}_i + \rho \bar{b}_{12}\right] & \text{if } i \in \alpha^* \\ 0 & \text{if } i \notin \alpha^* \end{cases} \tag{8}$$

Where, the expected number of clicks, $\left[\rho_i \bar{b}_i + \rho \bar{b}_{12}\right]$, is comprised of $\rho_i \bar{b}_i$ clicks by exclusive subscribers, and $\rho \bar{b}_{12}$ clicks by multihomers, who use only the platform's core service, regardless of e_i.

Platform prices in each expansion subgame are Nash equilibrium, such that:

$$p_i^* = \arg\max \pi_i\left(p_i|p_j^*, e_1, e_2, \bar{B}\right) \tag{9}$$

Platform i's equilibrium expected profits in an expansion subgame (e_1, e_2) are therefore $\pi_i^{e_1 e_2} \equiv \pi_i\left(p_i^*|p_j^*, e_1, e_2, \bar{B}\right)$.

The payoff matrix for the platform expansion game is then constructed using $(\pi_1^{e_1 e_2}, \pi_2^{e_1 e_2})$ for each subgame. We analyze the simultaneous move expansion game, deriving Nash Equilibrium expansion decisions (e_1^*, e_2^*), such that:

$$e_i^* = \arg\max \pi_i^{e_1 e_2}\left(e_i, e_j^*\right) \tag{10}$$

This simultaneous move game represents an environment where platforms' development efforts are kept secret until new services are launched.

Whenever there are multiple equilibria in the simultaneous move expansion game, we further analyze the sequential move game to examine which equilibrium would arise when platforms' development efforts are known. The equilibrium concept is then SGPE, and we consider both the case where the larger platform is the first mover, as well as the case where it is the follower.

2.5 Timeline

The timeline of the model is as follows:

1. Platforms make expansion decisions e_1 and e_2.

2. Final buyer partition $\bar{B}$ is determined, given buyers' choice rule and the level of mobility, $\beta^{e_1 e_2}$.

3. Platforms set prices per user click p_1 and p_2, given e_1, e_2, and $\bar{B}$, as well as advertisers choice rule.

4. Advertisers choose the platform(s) on which they place their ads, α^*, given $(e_i, p_i)_{i=1,2}$ and $\bar{B}$.

5. Platform profits are determined.

This is summarized in the following figure 1.

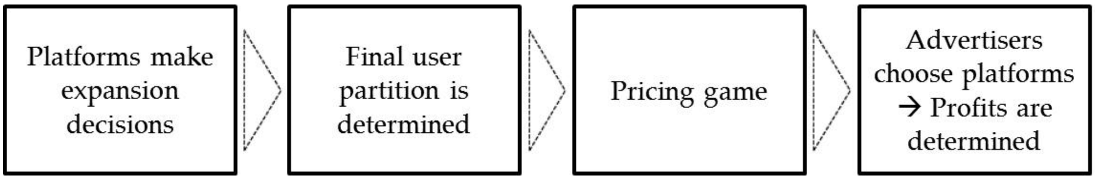

Figure 1: Timeline of the model

2.6 Market equilibrium

We define market equilibrium for the simultaneous move expansion game.

Definition 1 Market equilibrium *is a triplet* $\left\langle \bar{B}, \alpha^*, (e_i^*, p_i^*)_{i=1,2} \right\rangle$ *such that, given the initial buyer partition* B:

1. *Buyers' final partition* $\bar{B}$ *is determined by platforms' expansion decisions affecting* $\beta^{e_1 e_2}$, *and movers' choice rule.*

2. *Advertisers' platform choice is optimal, given platforms' expansion and pricing decisions, and the resulting buyer partition:* $\alpha^* = \arg\max_{\alpha \in A} \left\{ V \left(\alpha | \left(e_i, p_i\right)_{i=1,2}, \ \bar{B} \right) \right\}$.

3. *Platform pricing is Nash equilibrium, given platforms' expansion decisions, the resulting buyer partition, and advertisers' choice rule:* $p_i^* = \arg\max \pi_i \left(p_i | p_j^*, e_1, e_2, \bar{B} \right)$.

4. *Platforms' expansion decisions are Nash equilibrium in the expansion game, where profits in each expansion subgame are determined by 1-3:* $e_i^* = \arg\max \pi_i^{e_1 e_2} \left(e_i, e_j^* \right)$.

As noted, we will consider the sequential move expansion game as a means of equilibrium selection, whenever multiple equilibria arise in the simultaneous move game. The definition of market equilibrium will be similar, differing only in the equilibrium concept for optimal expansion decisions (item 4), which will be SGPE.

3 Analysis and main effects

Platform expansion increases the CTR for exclusive users, while changing the partition of users through its effect on user mobility. The change in buyer partition then affects platforms' pricing and profits. We hereby derive buyers' final partition, platforms' optimal prices per click, and optimal choice rules for expansion, while discussing these effects, which we label the *CTR effect*, the *quantity effect of mobility*, and the *price effect of mobility*.

3.1 The CTR effect of expansion

The *CTR effect* refers to the increase in CTR for exclusive platform users brought on by expansion. Specifically, expansion increases CTR for these users from the one-service CTR, ρ, to the two-service CTR, $(2\rho - \rho^2)$. The CTR effect is thus a positive quantity effect, which drives platforms towards expansion. It immediately follows that, absent mobility effects, when $\Delta\beta = 0$, both platforms will always expand and the equilibrium is (E, E).

It is important to note that the CTR effect decreases in magnitude as ρ increases.

3.2 Buyer partition and the quantity effect of mobility

Given a pair of expansion decisions (e_1, e_2), the final partition of buyers, $\bar{B}$, is such that:

$$\bar{b}_i = (1 - \beta^{e_1e_2})\, b_i \tag{11}$$

$$\bar{b}_{12} = (1 - \beta^{e_1e_2})\, b_{12} + \beta^{e_1e_2} \tag{12}$$

The *quantity effect of mobility* is the change in the partition of users, directly resulting from the change in mobility. When mobility increases, both platforms are left with smaller masses of exclusive users, but at the same time sell more clicks by multihomers. We summarize the quantity effect of increased mobility for platform i by focusing on the increase in the mass of multihomers, comprised of: (1) $\beta^{e_1e_2}b_i$ users lost by platform i - this increase in b_{12} comes at own "expense"; (2) $\beta^{e_1e_2}b_j$ users lost by the opponent platform j - this increase comes at j's "expense".

Given that platform i expands, own user loss creates a negative quantity effect, since the migrating users would generate a higher CTR as exclusive users than they do as multihomers. However, given that i has not expanded, migrating users are characterized by the same CTR whether they are exclusive users or multihomers. In this case there is no quantity effect for lost users. On the other hand, increased multihoming due to buyers lost by the rival platform always creates a positive quantity effect.

Further note that the quantity effect depends on platforms' relative size. Specifically, when platforms are asymmetric, the smaller platform benefits more from its rival loss than the larger platform.

This quantity effect is the direct effect of increased mobility. There further exists an indirect effect - the price effect of multihoming, discussed in the following subsection.

3.3 Optimal pricing and the price effect of mobility

We turn to platforms' optimal pricing in a given expansion subgame. Given (e_1, e_2) and the resulting $\bar{B}$, advertisers place ads on both platforms whenever $V^{12} \geq V^1, V^2, 0$, and choose a single platform i whenever $V^i > V^{12}, V^j, 0$. Solving $V^{12} \geq V^j$ we find that $\alpha^* = \{1, 2\}$

whenever $p_i \leq \hat{p}_i^{e_1e_2}$ for $i = 1, 2$, where:

$$\hat{p}_i^{e_1e_2} \equiv 1 - \frac{\rho^2\bar{b}_{12}}{\rho_i\bar{b}_i + \rho\bar{b}_{12}} \tag{13}$$

Furthermore, note that $V^i \geq V^j$ if and only if $p_i \leq \tilde{p}_i(p_j)$, where $\tilde{p}_i^{e_1e_2}(p_j) \equiv \frac{\left[\rho_i\bar{b}_i - \rho_j\bar{b}_j\right] + p_j\left[\rho_j\bar{b}_j + \rho\bar{b}_{12}\right]}{\rho_i\bar{b}_i + \rho\bar{b}_{12}}$.

It is easily verified that $\hat{p}_i^{e_1e_2} = \tilde{p}_i^{e_1e_2}(\hat{p}_j)$, thus $V^1 = V^2 = V^{12} = \rho^2\bar{b}_{12}$ for $p_i = \hat{p}_i^{e_1e_2}$, $i = 1, 2$.

Proposition 1 provides a characterization of the pricing equilibrium, and the resulting platform profits, for each pair of expansion decisions.

Proposition 1: *Given $\bar{B}$ platforms will set prices at $p_i^* = \hat{p}_i^{e_1e_2}$ for $i = 1, 2$, and thus platform profits in every subgame (e_1, e_2) are given by*

$$\pi_i^{e_1e_2} = \rho_i\bar{b}_i + \rho(1-\rho)\bar{b}_{12} \tag{14}$$

Proof: First note that $i \in \alpha^*$ for all $p_i \leq \hat{p}_i^{e_1e_2}$, and the profit maximizing price in this region is clearly $p_i = \hat{p}_i^{e_1e_2}$. We now consider $p_i > \hat{p}_i^{e_1e_2}$. Pricing at $p_i \geq 1$ leads to $i \notin \alpha^*$ and zero profits, and is not profit maximizing. Therefore assume $p_i \in (\hat{p}_i^{e_1e_2}, 1)$: if $p_j \in \left(\hat{p}_j^{e_1e_2}, 1\right)$ then only one platform is chosen by advertisers - assume that i is chosen, i.e. $V^i \geq V^j$. This implies zero profits for j, and a profitable deviation to $\hat{p}_j^{e_1e_2}$. Alternatively, if $p_i \in (\hat{p}_i^{e_1e_2}, 1)$ and $p_j = \hat{p}_j^{e_1e_2}$ then $V^j > V^i$ and thus $i \notin \alpha^*$, and i has a profitable deviation to $p_i = \hat{p}_i^{e_1e_2}$. We have thus shown that for any price $p_i \neq \hat{p}_i^{e_1e_2}$ there exists a profitable deviation to $p_i = \hat{p}_i^{e_1e_2}$. Nash equilibrium prices in a given subgame are thus $p_i^* = \hat{p}_i^{e_1e_2}$ for $i = 1, 2$. Substituting for $\hat{p}_i^{e_1e_2}$ in 8 yields the expression in 14 for platforms' profits in subgame (e_1, e_2).■

We write the first order derivatives of $\hat{p}_i^{e_1e_2}$ with respect to $\bar{b}_{12}$ and $\bar{b}_i$:

$$\frac{\partial\hat{p}_i^{e_1e_2}}{\partial\bar{b}_{12}} = -\frac{\rho^2\rho_i\bar{b}_i}{\left(\rho_i\bar{b}_i + \rho\bar{b}_{12}\right)^2} < 0 \tag{15}$$

$$\frac{\partial\hat{p}_i^{e_1e_2}}{\partial\bar{b}_i} = \frac{\rho_i\rho^2\bar{b}_{12}}{\left[\rho_i\bar{b}_i + \rho\bar{b}_{12}\right]^2} > 0 \tag{16}$$

Equilibrium prices per click decrease in the degree of multihoming, $\bar{b}_{12}$, and increase in the mass of exclusive users, $\bar{b}_i$. This is labeled the *price effect of mobility*, as increased mobility leads to increased multihoming and lower exclusivity, which result in lower prices. A higher degree of multihoming leads to lower prices, since advertisers place ads on both platforms in equilibrium, and thus reach multihomers through both platforms. This implies that equilibrium per click prices are constrained by the degree of multihomimg. Therefore, quite intuitively, market power in the model stems from platforms' exclusive users, and decreases in the degree of multihoming. The price effect of mobility is thus negative, whenever the change in mobility increases multihoming (and, at the same time, decreases exclusivity).

3.4 Optimal expansion rules

We proceed to derive conditions for platforms' expansion decisions. Expansion rules will clearly take into account the CTR effect, and both the price and quantity effects of mobility, and will depend on the relative magnitudes of these effects.

Given platform j's expansion decision, e_j, i will expand whenever $\pi_i^{Ee_j} \geq \pi_i^{\bar{E}e_j}$. (We assume that indifference is resolved by favoring expansion.) Solving yields, for $\rho < \bar{\rho}_{e_j}$:

$$e_i|_{e_j} = \begin{cases} E & \text{for } b_{12} \leq \tilde{b}^i_{e_j} \\ \bar{E} & \text{for } b_{12} > \tilde{b}^i_{e_j} \end{cases} \tag{17}$$

Whereas, for $\rho > \bar{\rho}_{e_j}$:

$$e_i|_{e_j} = \begin{cases} \bar{E} & \text{for } b_{12} < \tilde{b}^i_{e_j} \\ E & \text{for } b_{12} \geq \tilde{b}^i_{e_j} \end{cases} \tag{18}$$

Where $\bar{\rho}_{e_j}$ is given by:

$$\bar{\rho}_{e_j} = \frac{1 - \beta^{Ee_j} + \Delta\beta}{1 - \beta^{Ee_j} + 2\Delta\beta} \tag{19}$$

And the threshold $\tilde{b}^i_{e_j}$ depends on i, j, since $b_1 \geq b_2$, such that:

$$\tilde{b}^1_{e_2} = 1 + \delta \frac{(1-\rho)\left(1 - \beta^{Ee_2}\right) - \Delta\beta}{(1-\rho)\left(1 - \beta^{Ee_2}\right) - \Delta\beta + 2(1-\rho)\Delta\beta} \tag{20}$$

$$\tilde{b}^2_{e_1} = 1 - \delta \frac{(1-\rho)\left(1 - \beta^{Ee_1}\right) - \Delta\beta}{(1-\rho)\left(1 - \beta^{Ee_1}\right) - \Delta\beta + 2(1-\rho)\Delta\beta} \tag{21}$$

The intuition behind the above thresholds is as follows. When the CTR is relatively small, $\rho < \bar{\rho}_{e_j}$, the total quantity effect of CTR and mobility is positive, and therefore expansion is optimal whenever the price effect is not too large. This implies that the platforms will expand for relatively low levels of multihoming, $b_{12} \leq \tilde{b}^i_{e_j}$, and will not expand when the mass of multihomers is large, and the price effect of mobility overcomes the total quantity effect.

Alternatively, when the CTR is relatively large, $\rho > \bar{\rho}_{e_j}$, then both the price effect of mobility and the total quantity effect (of CTR and mobility) are negative. When this is the case, no-expansion is optimal for low levels of multihoming. However, it is possible that for high levels of b_{12}, the total negative effect will be weaker under expansion compared to no-expansion, and thus expansion will be optimal.

Using 17 and 18, and the thresholds defined in 19-21, equilibrium expansion decisions are derived.

4 Platform expansion equilibrium

4.1 Mobility increases with expansion

When expansion increases mobility, the price effect is clearly negative, due to the increase in multihoming. This is, of course, in addition to the always-positive CTR effect. The quantity effect is ambiguous, and will depend on platforms' relative size and expansion decisions.

We first examine equilibrium expansion decisions for initially symmetric platforms. The equilibrium is symmetric expansion for low levels of ρ due to dominance of the CTR effect, and symmetric no-expansion for high levels of ρ due to dominance of the price effect (both in dominant strategies). For mid-level CTRs, asymmetric equilibria arise, even though the platforms are symmetric at the outset. This is driven by the negative quantity effect arising from the rival platform's expansion. Specifically, when ρ is mid-ranged and the rival platform has not expanded, the negative effect of increasing mobility of own users is relatively small, and offset by the increase in rival users' mobility and the CTR effect. This implies that expansion is optimal, given the opponent has not expanded. However, when the opponent has expanded, a negative effect of own users' mobility is already in place, and the negative price effect of multihoming is stronger due to the rival's expansion. When this is the case, own expansion entails a quantity effect that cannot offset the now stronger price effect. Therefore, no-expansion is optimal, given the rival platform has expanded.

We now consider which of the asymmetric equilibria would arise for mid-level CTRs, when expansion decisions are made sequentially. Clearly, while both platforms are characterized by the same level of mobility, the expanding platform enjoys the favorable CTR effect, not enjoyed by its rival. It follows that in the sequential move game, the first mover will expand, and the follower will not.

The results are summarized in the following proposition 2, as well as in figure 2.

Proposition 2: *In the case of mobility that increases with expansion: asymmetric expansion may be an equilibrium, even when platforms are symmetric at the outset. Formally: when* $\Delta\beta > 0$ *and* $\delta = 0$*, there exist* $\bar{\rho}_E, \bar{\rho}_{\bar{E}}$*, such that* $0 < \bar{\rho}_E < \bar{\rho}_{\bar{E}} < 1$*:*

1. *For* $\rho \in (0, \bar{\rho}_E)$*: The equilibrium is* (E, E)*, i.e. symmetric expansion for low-level CTRs.*

2. *For* $\rho \in (\bar{\rho}_E, \bar{\rho}_{\bar{E}})$*: The equilibria are* $\left(E, \bar{E}\right)$ *and* $\left(\bar{E}, E\right)$*, i.e. asymmetric expansion for mid-level CTRs. When expansion decisions are made sequentially, the first mover will expand, and the follower will not: if platform* 1 *is the first mover the equilibrium is* $\left(E, \bar{E}\right)$*, and otherwise* $\left(\bar{E}, E\right)$*.*

3. *For* $\rho \in (\bar{\rho}_{\bar{E}}, 1)$*: The equilibrium is* $\left(\bar{E}, \bar{E}\right)$*, i.e. symmetric no-expansion for high-level CTRs.*

Proof: see appendix.

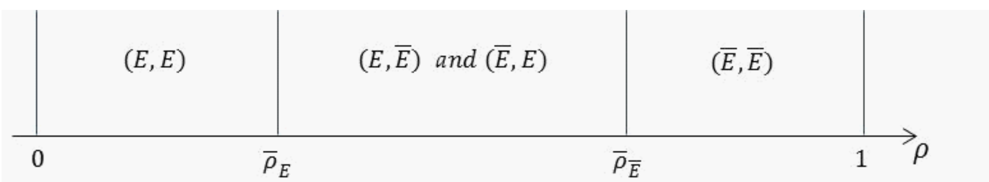

Figure 2: Equilibrium expansion decisions, as a function of ρ, for $\Delta\beta > 0$ and $\delta = 0$.

We turn to the case of asymmetric platforms, where the quantity effects of mobility are asymmetric. In this case, the smaller platform gains more from its rival's loss of users, and thus has a stronger incentive to increase mobility by expansion.

As in the case of symmetric platforms, the incentive to expand decreases as the CTR increases. When the CTR is small, expansion is, as before, a dominant strategy for both platforms. As the CTR increases, we identify a region where expansion is still a dominant strategy for the smaller platform, but the larger platform will not expand when the mass of multihomers is large. This is due to the larger platform's weaker quantity effect. Further increases in ρ, give rise to a region where, for high degrees of multihoming, only the smaller platform expands, while for low degrees of multihoming both types of asymmetric expansion equilibria may arise, and the intuition is the same as for the case of symmetric platforms. When the CTR is very large, the quantity effect of expansion is negative for both platforms. In this region, the larger platform will not expand, while the smaller platform expands only for large degrees of multihoming.

This equilibrium characterization is presented in proposition 3 and in the figure that follows.

Proposition 3: *In the case of mobility that increases with expansion: asymmetric and symmetric equilibria may both arise, when platforms are asymmetric at the outset. Formally: when* $\Delta\beta > 0$ *and* $\delta > 0$*, there exist* $\tilde{\rho}_E^1, \bar{\rho}_E, \bar{\rho}_{\bar{E}}$*, such that* $0 < \tilde{\rho}_E^1 < \bar{\rho}_E < \bar{\rho}_{\bar{E}} < 1$*:*

1. *For* $\rho \in \left(0, \tilde{\rho}_E^1\right)$*: The equilibrium is* (E, E)*, i.e. symmetric expansion for low-level CTRs.*

2. *For* $\rho \in \left(\tilde{\rho}_E^1, \bar{\rho}_E\right)$ *there exists* $\tilde{b}_E^1 \in (0, 1-\delta)$*: The equilibrium is* (E, E) *for* $b_{12} \leq \tilde{b}_E^1$ *and* $\left(\bar{E}, E\right)$ *otherwise.*

3. *For* $\rho \in (\bar{\rho}_E, \bar{\rho}_{\bar{E}})$ *there exist* $\tilde{b}_{\bar{E}}^1, \tilde{b}_E^2 \in (0, 1-\delta)$*: The equilibrium is* $\left(\bar{E}, E\right)$ *for* $b_{12} > \min\left\{\tilde{b}_{\bar{E}}^1, \tilde{b}_E^2\right\}$*, and otherwise* $\left(E, \bar{E}\right)$ *and* $\left(\bar{E}, E\right)$ *are both equilibria. For* $b_{12} \leq \min\left\{\tilde{b}_{\bar{E}}^1, \tilde{b}_E^2\right\}$ *and the case of sequential expansion decisions, the first mover will expand, and the follower will not: if platform 1 is the first mover the equilibrium is* $\left(E, \bar{E}\right)$*, and otherwise* $\left(\bar{E}, E\right)$*.*

4. *For* $\rho \in (\bar{\rho}_{\bar{E}}, 1)$ *there exists* $\tilde{b}_{\bar{E}}^2 \in (0, 1-\delta)$*: The equilibrium is* $\left(\bar{E}, \bar{E}\right)$ *for* $b_{12} \leq \tilde{b}_{\bar{E}}^2$ *and* $\left(\bar{E}, E\right)$ *otherwise.*

Proof: see appendix.

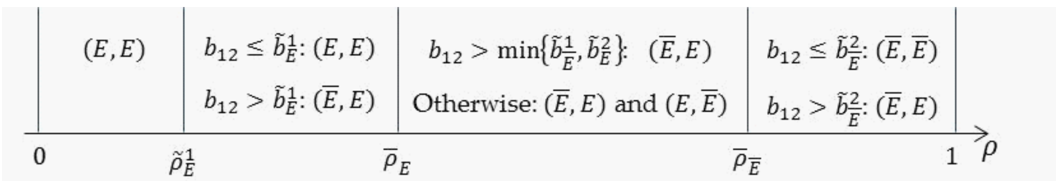

Figure 3a: Equilibrium expansion decisions as a function of ρ, for $\Delta\beta > 0$ and $\delta > 0$.

To further demonstrate the optimality of no-expansion when the degree of multihoming is large, we turn our attention to figure 3b. The figure graphs profits for platform 1 under expansion and no-expansion, as a function of the initial degree of multihoming, for relatively low CTRs, $\rho \in \left(\tilde{\rho}_E^1, \bar{\rho}_E\right)$. *Given* that a platform has expanded, profits for this platform are always decreasing in b_{12}, because under expansion, exclusive users have a higher CTR, and thus each user lost to multihoming lowers profits.[4]

Alternatively, *given* that a platform has not expanded and $\rho < 0.5$, platform profits are increasing in b_{12}. This is because, under no-expansion, exclusive and multihoming users have the same CTR, and for $\rho < 0.5$ the quantity effect of increased multihoming is stronger than the price effect. This implies that for $\rho \in \left(\tilde{\rho}_E^1, \bar{\rho}_E\right)$, there exists $\tilde{b}_E^1$, such that $\pi_1^{\bar{E}E} > \pi_1^{EE}$ for $b_{12} > \tilde{b}_E^1$, and so the larger platform will optimally not expand in this domain.

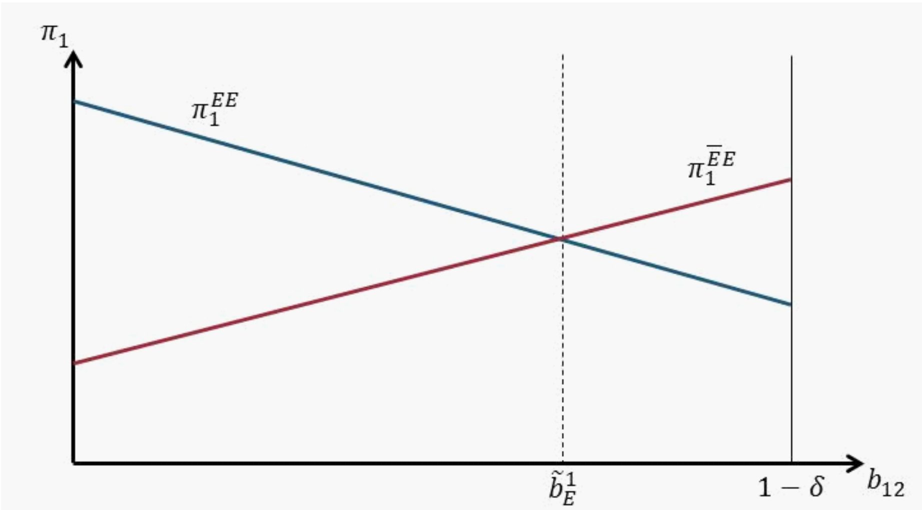

Figure 3b: Profits for platform 1 under expansion and no-expansion, as a function of b_{12}, for $\rho \in \left(\tilde{\rho}_E^1, \bar{\rho}_E\right)$.

4.2 Mobility decreases with expansion

When expansion decreases mobility, the price effect of expansion is positive, as is the quantity effect arising from decreased loss of exclusive users. The quantity effect resulting from the rival's user loss is negative. When the change in mobility is small, expansion is a dominant strategy for both platforms, due to the positive price effect, the positive quantity effect of reduced own-user loss, and the (always) positive CTR effect. However, when the change in mobility is large, the quantity effect resulting from the opponent's loss of subscribers becomes stronger, and no-expansion equilibria may arise.

The above intuition drives the following result, stated for symmetric platforms in proposition 4 and in figure 4.

[4] Mobility effects are irrelevant, as we focus on profits *given* expansion.

Proposition 4: *In the case of mobility that decreases with expansion: when platforms are symmetric at the outset the equilibrium is symmetric - either both platforms expand or both do not. Formally: when* $-0.5\beta^0 < \Delta\beta < 0$*, and* $\delta = 0$*, the equilibrium depends on the initial mobility* β^0 and on $|\Delta\beta|$:

1. *For* $\beta^0 \leq \frac{2}{3}$*: The equilibrium is* (E, E)*.*
2. *For* $\beta^0 > \frac{2}{3}$*: (a) When* $|\Delta\beta| \leq 1 - \beta^0$*, the equilibrium is* (E, E)*, i.e. small changes in mobility lead to symmetric expansion; (b) When* $1 - \beta^0 < |\Delta\beta| \leq 0.5\beta^0$*, the equilibria are* (E, E) *and* $(\bar{E}, \bar{E})$*, i.e. both symmetric expansion and no-expansion are possible for large changes in mobility. When expansion decisions are made sequentially, the SGPE is* $(\bar{E}, \bar{E})$*.*

Proof: see appendix.

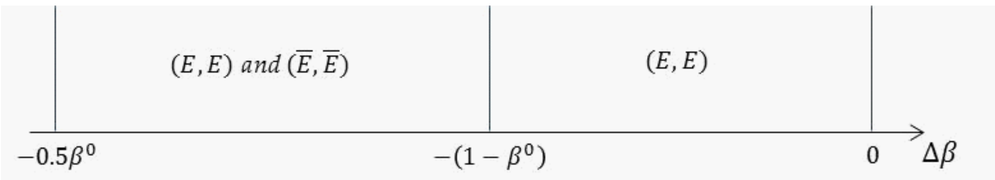

Figure 4: Equilibrium expansion decisions as a function of $\Delta\beta$, when $\Delta\beta < 0$, for $\delta = 0$, $\beta^0 > \frac{2}{3}$ (when $\beta^0 \leq \frac{2}{3}$ the left region does not exist)

When the platforms are asymmetric, the above intuition holds, but now there may be cases with an asymmetric expansion equilibrium due to the asymmetric quantity effect of mobility. Specifically, the smaller platform benefits from its rival's mobility, and may therefore not expand in equilibrium. This will occur when the degree of multihoming is large, and thus the price effect of expansion is relatively small.

The result is presented in proposition 5 and the following figure 5.

Proposition 5: *In the case of mobility that decreases with expansion: when platforms are asymmetric at the outset the equilibrium is may be either symmetric or asymmetric, and depends on both the mobility parameters,* β^0*,*$\Delta\beta$*, and the size of the multihoming group* b_{12}*. Formally, when* $-0.5\beta^0 < \Delta\beta < 0$*, and* $\delta > 0$*:*

1. *For* $\beta^0 \leq \frac{2}{3}$ *or* $\beta^0 > \frac{2}{3}$ *and* $|\Delta\beta| \leq 1 - \beta^0$*, there exists* $\tilde{b}^2_E \in (0, 1 - \delta)$*: The equilibrium is* (E, E) *for* $b_{12} \leq \tilde{b}^2_E$*, and* $(E, \bar{E})$ *otherwise.*
2. *For* $\beta^0 > \frac{2}{3}$ *and* $1 - \beta^0 < |\Delta\beta| \leq 0.5\beta^0$*, there exists* $\tilde{b}^1_{\bar{E}}, \tilde{b}^2_E \in (0, 1 - \delta)$*, such that* $\tilde{b}^1_{\bar{E}} < \tilde{b}^2_E$*: The equilibria are* (E, E) *and* $(\bar{E}, \bar{E})$ *for* $b_{12} \leq \tilde{b}^1_{\bar{E}}$*; The equilibrium is* (E, E) *for* $b_{12} \in \left(\tilde{b}^1_{\bar{E}}, \tilde{b}^2_E\right)$*, and* $(E, \bar{E})$ *otherwise. For* $b_{12} \leq \tilde{b}^1_{\bar{E}}$*, and sequential expansion decisions, the SGPE is either* (E, E) *or* $(\bar{E}, \bar{E})$*, depending on the levels of* $b_{12}, \rho, \beta^0, \Delta\beta$*.*

Proof: see appendix.

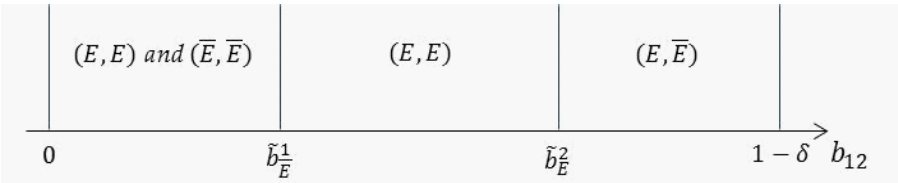

Figure 5: Equilibrium expansion decisions as a function of b_{12}, for $\delta > 0$, $\Delta\beta < -\left(1-\beta^0\right)$, and $\beta^0 > \frac{2}{3}$ (when $\Delta\beta > -\left(1-\beta^0\right)$ or $\beta^0 \leq \frac{2}{3}$the left region does not exist)

Note that the intuition for platform 2's expansion decisions for $b_{12} > \tilde{b}^1_{\overline{E}}$, is the same as presented in figure 3b and the preceding discussion.

5 Discussion

We have presented a game theoretic modeling framework for analysis of online platforms' expansion decisions, focusing on expansion into services already offered in the market. The analysis demonstrates that platforms may not expand in equilibrium, even though expansion is assumed to be costless, and increases the CTR for platform users. This is due to the endogenous user mobility in the model, which creates quantity and price effects to platform expansion.

We have analyzed the model under the assumption that migrating users become multi-homers, which could represent a positive network effect for the core services available at the outset, or a higher quality for these services. An alternative assumption is that migrating users prefer to singlehome, i.e. subscribing to a single platform, whenever at least one of the platforms expands.

Under this alternative assumption, movers continue to prefer subscription to two service over one, but multihome only when no platform expands. When one platform expands, it attracts all the movers in the market, and when both platforms expand, migrating buyers will subscribe to the platform which was larger at the outset. This is interpreted as a *platform-related positive network effect.*

In this case, when expansion raises the level of mobility, it decreases multihoming and creates a positive price effect for both platforms. The quantity effect is always positive for the larger platform, which will always choose to expand. The smaller platform, on the other hand, may not expand, as expansion increases loss of own users (exclusive users and multihomers), who migrate to the larger platform. The smaller platform's expansion decision, will depend on the CTR, on the initial mobility and the expected change in mobility, as well as on the degree of multihoming.

As in the base case examined, the equilibrium will depend on the relative sizes of the CTR effect, price effect, and quantity effect, where the first two are positive, and the third is negative. The smaller platform will not expand when the CTR is relatively large, when the change in mobility caused by expansion is large, or when the degree of multihoming is small. Market equilibrium may thus be either symmetric expansion, or asymmetric expansion where the larger platform expands and the smaller platform does not.

Considering the above alternative assumption regarding buyers' choice rule highlights the importance of endogenous user mobility in our modeling framework. When the level of mobility changes with the introduction of new services, expansion by one platform exerts strategic effects on its rival, directly through the change in user partition, and indirectly through prices. This feature leads to the various types of market equilibria that arise in the model.

Is it reasonable to assume that users' mobility changes with the number of services in the market? While extremely difficult to measure, it seems that mobility in online markets does change over time, whether it increases with platform compatibility (and thus decreases when compatibility decreases), or increases with the number of alternatives available to users in the market.

Appendix

Proof of proposition 2: For $\delta = 0$ the thresholds $\tilde{b}^i_{e_j} = 1$ for $i = 1, 2$. Furthermore, $\Delta\beta > 0$ implies that $0 < \bar{\rho}_E < \bar{\rho}_{\bar{E}} < 1$. We now specify decision rules and the resulting equilibrium for each domain of ρ:

1. For $\rho \in (0, \bar{\rho}_E)$: $e_i|_{e_j} = E$ whenever $b_{12} \leq 1$. $e_i = E$ is a dominant strategy for $i = 1, 2$, and the equilibrium is (E, E).

2. For $\rho \in (\bar{\rho}_E, \bar{\rho}_{\bar{E}})$: $e_i|_{\bar{E}} = E$ whenever $b_{12} \leq 1$, and $e_i|_E = \bar{E}$ whenever $b_{12} \leq 1$. Both $\left(E, \bar{E}\right)$ and $\left(\bar{E}, E\right)$ are thus equilibria. Comparing profits for the two asymmetric equilibria yields $\pi_1^{E\bar{E}} > \pi_1^{\bar{E}E}$ and $\pi_2^{\bar{E}E} > \pi_2^{E\bar{E}}$. This implies that profits are higher for the expanding platform. Therefore, in a sequential move expansion game the first mover will expand and the follower will not.

3. For $\rho \in (\bar{\rho}_{\bar{E}}, 1)$: $e_i|_{e_j} = \bar{E}$ whenever $b_{12} \leq 1$. $e_i = \bar{E}$ is a dominant strategy for $i = 1, 2$, and the equilibrium is $\left(\bar{E}, \bar{E}\right)$.

■

Proof of proposition 3: For $\delta > 0$, the initial degree of multihoming is $b_{12} < 1 - \delta$. We first derive conditions for $\tilde{b}^i_{e_j} < 1 - \delta$ for $i = 1, 2$.

$\tilde{b}^1_{e_2} < 1 - \delta$ whenever:

$$1 + \delta \frac{(1-\rho)\left(1-\beta^{Ee_2}\right) - \Delta\beta}{(1-\rho)\left(1-\beta^{Ee_2}\right) - \Delta\beta + 2(1-\rho)\Delta\beta} < 1 - \delta \tag{22}$$

This implies that for $\rho < \bar{\rho}_{e_2}$: $\tilde{b}^1_{e_2} < 1 - \delta \iff \rho > \tilde{\rho}^1_{e_2} \equiv \frac{1-\beta^{Ee_2}}{1-\beta^{Ee_2}+\Delta\beta}$, while for $\rho > \bar{\rho}_{e_2}$: $\tilde{b}^1_{e_2} < 1 - \delta \iff \rho < \tilde{\rho}^1_{e_2} \equiv \frac{1-\beta^{Ee_2}}{1-\beta^{Ee_2}+\Delta\beta}$.

$\tilde{b}^2_{e_1} < 1-\delta$ whenever:

$$1-\delta\frac{(1-\rho)\left(1-\beta^{e_1E}\right)-\Delta\beta}{(1-\rho)\left(1-\beta^{e_1E}\right)-\Delta\beta+2(1-\rho)\Delta\beta} < 1-\delta \tag{23}$$

This implies that for $\rho < \bar{\rho}_{e_1}$: $\tilde{b}^2_{e_1} < 1-\delta \iff \Delta\beta < 0$, while for $\rho > \bar{\rho}_{e_1}$: $\tilde{b}^2_{e_1} < 1-\delta \iff \Delta\beta > 0$.

Since $\Delta\beta > 0$: $\tilde{b}^2_{e_1} > 1-\delta$ for $\rho < \bar{\rho}_{e_1}$, and $\tilde{b}^2_{e_1} < 1-\delta$ for $\rho > \bar{\rho}_{e_1}$.

Furthermore, $\Delta\beta > 0$ implies that $0 < \tilde{\rho}^1_E < \tilde{\rho}^1_{\bar{E}} < \bar{\rho}_E < \bar{\rho}_{\bar{E}} < 1$. We now specify decision rules and the resulting equilibrium for each domain of ρ:

1. For $\rho \in \left(0, \tilde{\rho}^1_E\right)$: $e_i|_{e_j} = E$ whenever $b_{12} \le \tilde{b}^i_{e_j}$, and $\tilde{b}^i_{e_j} > 1-\delta$ for $i = 1,2$. $e_i = E$ is a dominant strategy for $i = 1,2$, and the equilibrium is (E,E).

2. For $\rho \in \left(\tilde{\rho}^1_E, \tilde{\rho}^1_{\bar{E}}\right)$ and $\left(\tilde{\rho}^1_{\bar{E}}, \bar{\rho}_E\right)$: $e_i|_{e_j} = E$ whenever $b_{12} \le \tilde{b}^i_{e_j}$. Since $\tilde{b}^2_{e_i} > 1-\delta$, $e_2 = E$ is a dominant strategy. $\tilde{b}^1_E < 1-\delta$, such that platform 1's strategy depends on b_{12}. The equilibrium is (E,E) for $b_{12} \le \tilde{b}^1_E$ and $\left(\bar{E}, E\right)$ for $b_{12} > \tilde{b}^1_E$.

3. For $\rho \in (\bar{\rho}_E, \bar{\rho}_{\bar{E}})$: $e_i|_{\bar{E}} = E$ whenever $b_{12} \le \tilde{b}^i_{\bar{E}}$, while $e_i|_E = E$ whenever $b_{12} \ge \tilde{b}^i_E$ Since $\tilde{b}^1_{\bar{E}} < 1-\delta$, $\tilde{b}^1_E > 1-\delta$, $\tilde{b}^2_{\bar{E}} > 1-\delta$, and $\tilde{b}^2_E < 1-\delta$ - $e_1|_E = \bar{E}$, and $e_2|_{\bar{E}} = E$, while $e_1|_{\bar{E}}$ and $e_2|_E$ depend on b_{12}. For $b_{12} > \min\left\{\tilde{b}^1_{\bar{E}}, \tilde{b}^2_E\right\}$ the equilibrium is $\left(\bar{E}, E\right)$, and otherwise $\left(\bar{E}, E\right)$ and $\left(E, \bar{E}\right)$ are both equilibria. For $b_{12} \le \min\left\{\tilde{b}^1_{\bar{E}}, \tilde{b}^2_E\right\}$, comparing profits for the two asymmetric equilibria yields $\pi^{E\bar{E}}_1 > \pi^{\bar{E}E}_1$ and $\pi^{\bar{E}E}_2 > \pi^{E\bar{E}}_2$. This implies that profits are higher for the expanding platform. Therefore, in a sequential move expansion game the first mover will expand and the follower will not.

4. For $\rho \in (\bar{\rho}_{\bar{E}}, 1)$: $e_i|_{e_j} = E$ whenever $b_{12} \ge \tilde{b}^i_{e_j}$. Since $\tilde{b}^1_{e_2} > 1-\delta$, $e_1 = \bar{E}$ is a dominant strategy. $\tilde{b}^2_{\bar{E}} < 1-\delta$, such that platform 2's strategy depends on b_{12}. The equilibrium is $\left(\bar{E}, \bar{E}\right)$ for $b_{12} \le \tilde{b}^2_{\bar{E}}$ and $\left(\bar{E}, E\right)$ for $b_{12} > \tilde{b}^2_{\bar{E}}$.

■

Proof of proposition 4: For $\delta = 0$ the thresholds $\tilde{b}^i_{e_j} = 1$ for $i = 1,2$. For $-0.5\beta^0 < \Delta\beta < 0$, there are two possible cases, depending on β^0 and $|\Delta\beta|$:

1. For $\beta^0 \le \frac{2}{3}$: $-0.5\beta^0 > -\left(1-\beta^0\right)$, thus for all $-0.5\beta^0 < \Delta\beta < 0$ and $\bar{\rho}_{\bar{E}}, \bar{\rho}_E > 1$. This implies $\rho < \bar{\rho}_{\bar{E}}, \bar{\rho}_E\ \forall\rho \Rightarrow e_i|_{e_j} = E$ for $b_{12} \le 1$. $e_i = E$ is a dominant strategy for $i = 1,2$, and the equilibrium is (E,E).

2. For $\beta^0 > \frac{2}{3}$: (a) When $|\Delta\beta| \le 1-\beta^0$: the analysis is the same as in 1; (b) When $1-\beta^0 < |\Delta\beta| \le 0.5\beta^0$, the thresholds for ρ are $\bar{\rho}_{\bar{E}} < 0, \bar{\rho}_E > 1$. This implies $\bar{\rho}_{\bar{E}} < \rho < \bar{\rho}_E$ $\forall\rho \Rightarrow e_i|_E = E$ for $b_{12} \le 1$, and $e_i|_{\bar{E}} = \bar{E}$ for $b_{12} \le 1$. The equilibria are (E,E) and $\left(\bar{E},\bar{E}\right)$. Comparing profits for the two symmetric equilibria yields $\pi^{\bar{E}\bar{E}}_i > \pi^{EE}_i$. Therefore, in a sequential move expansion game the SGPE is $\left(\bar{E},\bar{E}\right)$.

■

Proof of proposition 5: For $\delta > 0$ and $-0.5\beta^0 < \Delta\beta < 0$:

1. For $\beta^0 \leq \frac{2}{3}$, or $\beta^0 > \frac{2}{3}$ and $|\Delta\beta| \leq 1 - \beta^0$: the thresholds for ρ are $\bar{\rho}_{\bar{E}}, \bar{\rho}_E > 1$. This implies that $\rho < \bar{\rho}_{\bar{E}}, \bar{\rho}_E \; \forall\rho \Rightarrow e_i|_{e_j} = E$ for $b_{12} \leq \tilde{b}^i_{e_j}$, where the thresholds for b_{12} are $\tilde{b}^1_{e_2} > 1 - \delta$, $\tilde{b}^2_{\bar{E}} < \tilde{b}^2_E < 1 - \delta$. $e_1 = E$ is a dominant strategy, and the equilibrium is (E, E) for $b_{12} \leq \tilde{b}^2_E$, and $\left(E, \bar{E}\right)$ otherwise.

2. For $\beta^0 > \frac{2}{3}$ and $1 - \beta^0 < |\Delta\beta| \leq 0.5\beta^0$: the thresholds for ρ are $\bar{\rho}_{\bar{E}} < 0$, $\bar{\rho}_E > 1$. This implies that $\bar{\rho}_{\bar{E}} < \rho < \bar{\rho}_E \; \forall\rho \Rightarrow e_i|_E = E$ for $b_{12} \leq \tilde{b}^i_E$, and $e_i|_{\bar{E}} = \bar{E}$ for $b_{12} \leq \tilde{b}^i_{\bar{E}}$, where the thresholds for b_{12} are $\tilde{b}^1_{\bar{E}}, \tilde{b}^2_E < 1 - \delta$, and $\tilde{b}^1_E, \tilde{b}^2_{\bar{E}} > 1 - \delta$. For $b_{12} \leq \tilde{b}^1_{\bar{E}}$ both (E, E) and $\left(\bar{E}, \bar{E}\right)$ are equilibria; for $b_{12} \in \left(\tilde{b}^1_{\bar{E}}, \tilde{b}^2_E\right)$ the equilibrium is (E, E), and for $b_{12} \in \left(\tilde{b}^2_E, 1 - \delta\right)$ the equilibrium is $\left(E, \bar{E}\right)$. Lastly, for $b_{12} \leq \tilde{b}^1_{\bar{E}}$, and sequential expansion decisions: both $\pi_i^{\bar{E}\bar{E}} > \pi_i^{EE}$ and $\pi_i^{\bar{E}\bar{E}} < \pi_i^{EE}$ are possible, and the direction of the equilibrium depends on $b_{12}, \rho, \beta^0, \Delta\beta$.

■

References

Amelio A. and B. Jullien,"Tying and freebies in two-sided markets", *International Journal of Industrial Organization*, 2012, 30(5), pp. 436–446.

Armstrong, M., "Competition in Two-Sided Markets", *The Rand Journal of Economics*, 2006, 37(3), pp. 668-691.

Behringer, S.and L. Filistrucchi, "Price Wars in Two-Sided Markets: The case of the UK Quality Newspaper Industry", *working paper.*

Caillaud, B. and Jullien, B., "Competing cybermediaries", *European Economic Review*, 2001, 45(4-6)., pp. 797-808.

Caillaud, B. and Jullien, B., "Chicken and egg: competition among intermediation service providers", *The RAND Journal of Economics*, 2003, 34(2), pp. 309-328.

Casadesus-Masanell, R. and F. Ruiz-Aliseda, "Platform competition, compatibility, and social efficiency," *IESE Research Papers*, 2009, D/798.

Choi, J.P., "Tying in Two-Sided Markets with Multi-Homing," *Journal of Industrial Economics*, 2010, 58(3), pp. 607–626.

Gabszewicz, J., D. Laussel, and N. Sonnac, "Press advertising and the ascent of the 'Pensee Unique'," *European Economic Review*, 2001, 45(4-6), pp. 641-651.

Gabszewicz, J., D. Laussel, and N. Sonnac, "Press Advertising and the Political Differentiation of Newspapers," *Journal of Public Economic Theory*, 2002, 4(3), pp. 317-334.

Hagiu, A., "Two-Sided Platforms: Product Variety and Pricing Structures", *Journal of Economics & Management Strategy*, 2009, 18(4), pp. 1011-1043.

Halaburda, H. and Y. Yehezkel, "Platform Competition under Asymmetric Information", *American Economic Journal: Microeconomics*, forthcoming.

Kaiser U. and J. Wright, "Price structure in two-sided markets: Evidence from the magazine industry", *International Journal of Industrial Organization*, 2006, 24, pp. 1– 28.

Rochet, J.C. and J. Tirole, "Platform Competition in Two-Sided Markets", *Journal of the European Economic Association*, 2003, 1(4), pp. 990-1029.

Rochet, J.C. and J. Tirole, "Two-Sided Markets: A Progress Report", *The RAND Journal of Economics*, 2006, 35(3), pp. 645-667.