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Abstract

We develop a model of dynamic platform formation under positive platform externalities. Users can switch between an incumbent and entrant platforms, switching opportunities arise stochastically and users can choose whether to accept or reject an opportunity to switch. For homogeneous users, we characterize the incumbency advantage implied by a given equilibrium realization of the switching process. For linear utility, incumbency advantage increases in the mean and dispersion of the incumbent's share during the switching process, which captures the momentum and coordination of the process. Heterogeneity in preferences may lead some users to delay their switching or never switch at all. Assuming that switching opportunities arrive according to a Poisson process, users switch to the entrant platform if the average preference favors the entrant and if preferences are not too polarized.

Keywords: platform Formation, Migration, Standardization and Compatibility, Industry Dynamics

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1 Introduction

The utility of moving into a city or joining a telecommunications platform depends crucially on positive consumption externalities generated by other participants in that same platform. A large body of economic theory (for instance, Farrell and Klemperer [2007]) predicts that such markets feature large incumbency advantages since entrants have an initial platform value of zero. This paper emphasizes a particular source of incumbency advantage: the incentive of users to free ride on one another by delay their time of switching until users have increased the entrant's platform value.

We assume that users receive opportunities to switch from an incumbent to an entrant according to a stochastic process. For instance, when deciding whether to move into a new city, users may need to wait for an available apartment. Users can thenaccept or forego this opportunity, with foregoing users continuing to recieve additional opportunities according to the same stochastic process. We allow for the arrival of switching opportunity to vary with time and to depend on the number of users who have already switched (for instance, due to a supply response to a large or low demand by switching consumers).

If users are homogeneous, the ideal time to switch is when the platform value of the entrant and the incumbent are equal, and an opportunity to switch is always accepted after the point. Switching earlier than this point implies foregoing some of the incumbent's platform value, but foregoing such an early opportunity to switch might imply foregoing some of the entrant's platform values, if one's next opportunity to switch materializes after half the users have already switched. Thus, users have an incentive to free-ride on one another by foregoing early switching opportunities, but this is curbed by the threat of finding oneself in a low platform value incumbent for a long period.

We associate incumbency advantage with the level of utility that the entrant must provide in order for users to accept early switching opportunities. We then show that incumbency advantage is closely related to the equilibrium path of the incumbent's share, denoted $h(t)$ at instant t. When utility is linear in the number of users in one's platform, incumbency advantage increases with the mean and dispersion of the incumbent's share during the transition period. The mean $\mathbb{E}[h(t)]$ captures the momentum of the equilibrium switching process: it is large if switching is slow at first but speeds up over time. This increases incumbency advantage because users who

forego early on have a high probability of switching alongside other users later, when the process is occurring at a fast pace. The dispersion $\mathbb{V}\left[h\left(t\right)\right]$ captures the coordination of switching opportunities. The larger is this dispersion, the more frequently the incumbent's share takes values close to 1 and 0 in equilibrium, that is, the more does the process allow for a large mass of users to switch in a short period of time. When this is the case, users again foresee the opportunity of switching alongside a large number of other users, and therefore tend to forego early switching opportunities.

We then allow for users to differ in their preferences over the two platforms and assume that switching opportunities arise according to a homogeneous Poisson process. Users that prefer the incumbent may initially forego their switching opportunities and accept them later when the entrant's platform value is large enough. An equilibrium where all users switch to the entrant can be sustained if the average preference in the economy favors the entrant. However, the dispersion in preferences increases incumbency advantage and, if preferences are sufficiently polarized, a split market with two platforms may emerge in equilibrium. Indeed, decreasing the dispersion of preferences reduces makes it easier to sustain an equilibrium where all users switch to the entrant.

The rest of the paper is organized as follows. Section 2 introduces the key ingredients of our model and relates incumbency advantage to the equilibrium switching process when users are homogeneous. Section 3 considers the case of users with heterogeneous preferences between platforms. Section 4 relates our model to the existing literature, and Section 5 concludes.

2 Homogeneous users Benchmark

2.1 Setup

We consider two platforms, an incumbent and an entrant, and a continuum of mass 1 of users. Time is continuous, $t \in \mathbb{R}_+$. At $t = 0$, all users are participating in the incumbent and entry can occur. There are positive consumption externalities: if the mass of users in a platform is $x \geq 0$, instantaneous utility of all users is $u\left(x\right)$ in the incumbent, and $u\left(x\right) + k$ in the entrant. We assume $u\left(x\right) : [0, 1] \mapsto [0, U]$ is continuously differentiable, strictly increasing, strictly concave, and $u\left(0\right) = 0$. The parameter $k \in \mathbb{R}$ measures the additional utility obtained in the entrant and will be

the measure of incumbency advantage. It can be understood, for instance, as the difference in platform qualities. We assume the market is covered and that users single-home (participate in only one platform).[5]

There is an exogenous technology according to which users receive opportunities to switch from the incumbent to the entrant. We allow this technology to evolve over time and to depend on the aggregate decisions of users. If the mass of users in the incumbent platform at instant t is $h(t)$, then the switching technology process prescribes that a density $-\frac{d\hat{h}(t,h(t))}{dt} \geq 0$ of users, chosen randomly from among those still in the incumbent, are given the opportunity to switch to the entrant at instant t. Thus, if users always accept, the equilibrium path of the incumbent's share satisfies $h(t) = \int_0^t -\frac{d\hat{h}(t,h(t))}{dt} dt \in [0,1]$. The earliest time at which all users have switched to the entrant is T ($t \geq T \Leftrightarrow h(t) = 0$). If $h(t)$ asymptotes towards $h(t) = 0$, we denote $T = \infty$.[6]

The switching technology can have multiple interpretations. Users may need to wait for a physical vacancies in a city, or users may have limited attention and only consider switching platforms with a frequency described by $-\frac{d\hat{h}(t,h(t))}{dt}$. In this case, users may consider switching more or less frequently when the mass of users at the entrant grows large.

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[5]This is a reasonable assumption for platforms like neighborhoods, telecommunications platforms and operating systems, but arguably less so in the case of social platforms. However, for those platforms, one's contribution to platform value and utility from participation depend on how much time and attention one devotes to each platform, these being finite resources for each user. If re-optimizing the allocation of one's attention between platforms is costly, it may be a reasonable approximation that users focus largely on the platforms where they expect to obtain the highest utility. In practice, it seems that few people split their time approximately evenly between multiple online social platform, unless they serve markedly different purposes (for instance, a dating website and a professional platform).

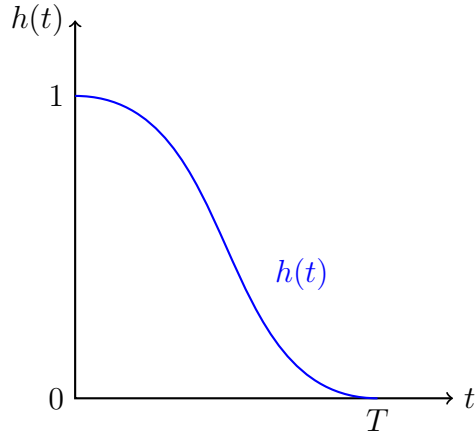[6]In this case, we assume $\int_0^\infty h(t) dt$ converges.

Figure 1: A typical equilibrium switching process $h(t)$.

Users discount the future at rate r. Then, in a candidate equilibrium where the mass in the incumbent evolves according to $h(t)$, the utility of a user that switches at time $t = t^\star$ is

$$\int_0^{t^\star} u(h(t)) e^{-rt} dt + \int_{t^\star}^\infty [u(1 - h(t)) + k] e^{-rt} dt$$

Intuitively, users are less likely to switch platforms if they are impatient (r large), since the utility lost during the transition between platforms is dearer relative to the the eventual benefit of coordinating on the entrant platform.[7] Therefore, impatience mechanically increases incumbency advantage. For this reason, we will focus on the limit as $r \to 0$, where users are perfectly patient. By doing so, we are focusing on sources of incumbency advantage other than this mechanical effect.

The action set of users faced with a switching opportunity is $\{0, 1\}$, where 1 indicates accepting and 0 indicates foregoing the opportunity. We use the following definitions.

Definition 1. A switching equilibrium is a Sub-Game Perfect Nash equilibrium where, for every user, there is an instant $t^\star$ such that an opportunity to switch is accepted for all $t \geq t^\star$.

Definition 2. For a given a candidate switching equilibrium where mass in the incumbent evolves according to $h(t)$, $k^\star$ is the minimum value of k for which there is

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[7]This is illustrated in Example 4.

no profitable deviation.

Notice that, if $k > u(1)$, switching is a dominant strategy for every user. By the same logic, if $k < -u(1)$, it is a dominant strategy never to switch. We therefore focus on the more interesting case where $-u(1) \leq k \leq u(1)$.

We further assume that the decision to switch to the entrant is a weakly increasing function of the number of users who have previously switched, $1 - h(t)$. This is especially reasonable in a setting where users are atomistic, there are no coordination mechanisms and users can only switch platforms when given the opportunity to do so. This is formalized as follows.

Assumption 1. *User strategies are a function $s(1 - h(t)) : [0,1] \mapsto \{0,1\}$ such that $x \geq \widetilde{x} \Rightarrow s(x) \geq s(\widetilde{x})$.*

Since users are homogeneous and strategies are monotonic in $h(t)$, it is the users who are given the opportunity of switching at $t = 0$ that have the highest incentive to deviate.

Lemma 1. *User strategies are indicators functions ($\exists S : s(1 - h(t)) = 1 \Leftrightarrow 1 - h(t) \geq S$). Moreover, for a candidate equilibrium where the mass in the incumbent evolves according to $h(t)$, there is no profitable deviation if and only if users accept to switch at $t = 0$.*

Proof. With a binary action space, an increasing function $s(1 - h(t))$ must imply that there exists some S such that $1 - h(t) < S \Rightarrow s(1 - h(t)) = 0$ and $1 - h(t) \geq S \Rightarrow s(1 - h(t)) = 1$. We normalize $s(S) = 1$ without loss of generality.

An equilibrium exists if and only if there are no profitable deviations by users. By Assumption 1, incentives to switch at t decrease in $h(t)$. By symmetry between users, the incentive to deviate is highest when no users have switched yet, so if there is no profitable deviation at $t = 0$, there is also none for $t > 0$. By the one-shot deviation principle (Blackwell [1965]), we can consider a candidate switching equilibrium where each user takes an opportunity to switch when it is presented, and consider the incentive of a user to deviate at $t = 0$ but follow the candidate equilibrium path for $t > 0$. $\square$

2.2 Results

We are now ready to discuss how the equilibrium switching process $h(t)$ affects incumbency advantage.

Proposition 1. *Define $\Delta(h(t)) \equiv u(h(t)) - u(1 - h(t))$. In the limit as $r \to 0$, the switching process $1 - h(t)$ is a switching equilibrium if and only if $k \geq \frac{\int_0^\infty h(t)\Delta(h(t))dt}{\int_0^\infty h(t)dt}$. If $T < \infty$, this can be expressed as $k \geq \mathbb{E}[\Delta(h(t))] + \frac{\mathrm{Cov}[\Delta(h(t)),h(t)]}{\mathbb{E}[h(t)]}$.*

Proof. By Lemma 1, we need only consider the incentives of a user given the opportunity to switch at $t = 0$. Switching at $t = 0$ yields $\int_0^\infty (u(1 - h(t)) + k)e^{-rt}dt$. A user forgoing a switch at $t = 0$ and switching when she is next given the chance (that is, following the equilibrium path after the deviation at $t = 0$), finds herself in the incumbent platform with probability $h(t)$, and therefore obtains

$$\int_0^\infty \{h(t)u(h(t)) + (1 - h(t))(u(1 - h(t)) + k)\}e^{-rt}dt.$$

This deviation is not profitable when the first integral is weakly greater than the second. Letting $r = 0$, this condition becomes

$$k\int_0^\infty h(t)\,dt \geq \int_0^\infty h(t)[u(h(t)) - (u(1 - h(t)))]\,dt$$

The integrals converge in the limit when $r = 0$ because $u(x)$ and $h(t)$ are both bounded and $h(t) \to 0$, by the assumption that $\int_0^\infty h(t)\,dt$ converges ($h(t)$ has "thin tails"). This then becomes

$$k \geq \lim_{T\to\infty} \frac{T\mathbb{E}[h(t)\Delta(h(t))]}{T\mathbb{E}[h(t)]} = \frac{\mathbb{E}[h(t)\Delta(h(t))]}{\mathbb{E}[h(t)]}$$

Using the identity $\mathbb{E}[XY] = \mathbb{E}[X]\mathbb{E}[Y] + \mathrm{Cov}[X,Y]$ yields the result. $\qquad\square$

The term $\Delta(h(t))$ measures how much, on average, utility in the incumbent platform is greater than utility in the new platform. This is the expected benefit from switching at a random instant t. Then, $\frac{d\Delta}{dt} = u'(h(t))h'(t) + u'(1 - h(t))h'(t) < 0$. Moreover, we have

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$$\Delta\left(h\left(t\right)\right) = \begin{cases} u\left(1\right) & ,h = 1 \Leftrightarrow t = 0 \\ 0 & ,h\left(t\right) = \frac{1}{2} \\ -u\left(1\right) & ,h = 0 \Leftrightarrow t = T \end{cases}$$

Then, if the process $h\left(t\right)$ is linear, $\Delta\left(h\left(t\right)\right)$ will be symmetric about the point $h\left(t\right) = \frac{1}{2} \Leftrightarrow t = \frac{T}{2}$, and $\mathbb{E}\left[\Delta\left(h\left(t\right)\right)\right] = 0$. However, if $h\left(t\right)$ is strictly concave, users switch rapidly in the latter half of the process so, on average over time, utility is higher in the incumbent: $\Delta\left(h\left(t\right)\right) = 0 \Leftrightarrow t > \frac{T}{2}$ and $\mathbb{E}\left[\Delta\left(h\left(t\right)\right)\right] > 0$. Conversely, if $h\left(t\right)$ is convex, $\Delta\left(h\left(t\right)\right) = 0 \Leftrightarrow t < \frac{T}{2}$ and $\mathbb{E}\left[\Delta\left(h\left(t\right)\right)\right] < 0$.

The term $\text{Cov}\left[\Delta\left(h\left(t\right)\right), h\left(t\right)\right]$ captures the extent to which this surplus from switching is likely to occur at a time when when the user finds herself in the incumbent and is therefore likely to actually benefit from the surplus. Notice that $\Delta\left(h\left(t\right)\right)$ is increasing in $h\left(t\right)$, so $\text{Cov}\left[\Delta\left(h\left(t\right)\right), h\left(t\right)\right] > 0$. Intuitively, the advantage of switching to the entrant (k) must exceed the benefit of participating in the incumbent.

While intuitive, Proposition 1 is hard to interpret because a general function $u\left(x\right)$ has a changing level of marginal utility for platform values, which complicates the tradeoff faced by users through time. In the following corollary, we consider the case of a linear utility function, where marginal utility for consumption externalities is fixed, which allows us to isolate the two key forces determining incumbency advantage.

Corollary 1. *If $u\left(x\right) = vx$, then tipping occurs when $\frac{k+v}{2v} \geq \frac{\int_0^\infty \left(h(t)\right)^2 dt}{\int_0^\infty h(t) dt}$. If $T < \infty$, this condition is equivalent to $\frac{k+v}{2v} \geq \mathbb{E}\left[h\left(t\right)\right]\left(1 + \frac{\mathbb{V}[h(t)]}{\mathbb{E}[h(t)]^2}\right)$.*

Proof. Following the proof of Proposition 1, a user is willing to accept the opportunity to switch at $t = 0$ if and only if

$$0 \geq \int_0^\infty h\left(t\right)\left\{vh\left(t\right) - v\left(1 - h\left(t\right)\right) - k\right\} dt = \int_0^\infty h\left(t\right) 2vh\left(t\right) dt - \left(k + v\right)\int_0^\infty h\left(t\right) dt$$

which yields the first inequality. The second equality can be obtained from Proposition 1 by using $\mathbb{E}\left(\Delta\left(h\left(t\right)\right)\right) = 2v\mathbb{E}\left[h\left(t\right)\right] - v$ and $\text{Cov}\left[\Delta\left(h\left(t\right)\right), h\left(t\right)\right] = 2v\mathbb{V}\left[h\left(t\right)\right]$. $\square$

Notice that, if $\frac{k+v}{2v} > 1 \Leftrightarrow k > v$, it is a dominant strategy for any user to switch independently of the switching technology of the behavior of other users. Similarly,

if $\frac{k+v}{2v} < 0 \Leftrightarrow k < -v$, then being alone in the incumbent is a dominant strategy. Notice also that $1 + \frac{\mathbb{V}[h(t)]}{\mathbb{E}[h(t)]^2}$ is a dimensionless factor multiplying $\mathbb{E}[h(t)]$, which is in units of time.

The term $\mathbb{E}[h(t)]$ captures the momentum of the equilibrium switching process. Suppose $T < \infty$ and consider a switching process where consumers switch slowly at first but increasingly fast as time goes on (that is, there is positive momentum). For instance, this can occur if the entrant does not build capacity until some users have already switched. Then, mass in the incumbent decreases slowly at first but eventually speeds up, $h(t)$ concave, $\mathbb{E}[h(t)]$ is large and incumbency advantage is large. This because there is a large probability that a user who foregoes a switching opportunity is given another opportunity to switch in a coordinated way with several other users once momentum builds up. A similar reasoning explains why process that start off quickly but eventually slow down (that is, there is negative momentum), leads to $h(t)$ convex, $\mathbb{E}[h(t)]$ small and small incumbency advantage. The term $\mathbb{E}[h(t)]$ can also be thought of as capturing whether, on average, switching opportunities occur early or late in the switching process.

The term $\frac{\mathbb{V}[h(t)]}{\mathbb{E}[h(t)]^2}$ captures, for a average timing of switching opportunities, how coordinated these opportunities. Intuitively, the larger is $\mathbb{V}[h(t)]$, the more often $h(t)$ takes values close to its extreme, 0 and 1. If this is the case, then $h(t)$ must transition from 1 to 0 in a relatively short period of time, which implies that a large mass of users switch in a coordinated way. Clearly, if users foresee a large probability of a coordinated switch, they will have a greater incentive to accept early switching opportunities.

The following examples help clarify the role of $\mathbb{V}[h(t)]$ and $\mathbb{E}[h(t)]$.

Example 1. For any linear process $1 - h(t) = at$, we have $T = 1/a$. Then $\mathbb{E}[h(t)] = \mathbb{E}[at] = \frac{1}{a} \int_0^{1/a} at\, dt = \frac{1}{2}$. Since the process is linear, there is no positive or negative momentum, so $\mathbb{E}[h(t)]$ takes its intermediate value $\frac{1}{2}$. This is independent of a, the speed of switching. Moreover, $\mathbb{V}[h(t)] = \mathbb{V}[at] = \frac{1}{a} \int_0^{1/a} \left(at - \frac{1}{2}\right)^2 dt = \frac{1}{12}$. Thus, both mean and variance are constant for a linear switching process. We can then use Corollary 1 to show that a linear switching process is a switching equilibrium only if $k \geq \frac{v}{3}$.

Example 2. Consider the process where $T = 1$ and $h(t) = \begin{cases} 1 - at & , 0 < t < \frac{1}{2} \\ a(1-t) & , \frac{1}{2} \leq t < 1 \end{cases}$,

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for $a \in (0, 1)$. We have $\mathbb{E}[h(t)] = \frac{1}{2}$ for all values of a. Moreover, we have $\mathbb{V}[h(t)] = \frac{3+a(a-3)}{12}$ increasing in a. Here, a increases the amount of perfectly coordinated switching at $t = \frac{1}{2}$ and thus increases incumbency advantage.

The following proposition shows that incumbency advantage can be positive as well as negative, that is, there can be excessive entry.

Proposition 2. *Consider the equilibrium switching process $h(t) = 1 - t^a$. Then we have $\lim_{a \to 0} k^\star = -1$ and $\lim_{a \to \infty} k^\star = 1$.*

Proof. We have $T = 1$. Then $\mathbb{E}[h(t)] = 1 - \mathbb{E}[t^a] = 1 - \int_0^1 t^a dt = \frac{a}{1+a}$, and $\mathbb{V}[h(t)] = \int_0^1 \left(1 - t^a - \frac{a}{1+a}\right)^2 dt = \frac{a^2}{(1+a)^2(1+2a)}$. As $a \to \infty$, all users switch at $t = 1$, $\lim_{a \to \infty} E[h(t)] = 1$ and $\lim_{a \to 0} \mathbb{V}[h(t)] = 0$. As $a \to 0$, all users switch at $t = 0$, $\lim_{a \to 0} E[h(t)] = 0$ and $\lim_{a \to \infty} \mathbb{V}[h(t)] = 0$. $\square$

This proposition emphasizes that the equilibrium switching process can distort user behavior in two directions. On the one hand, when there is strong positive momentum, the incentive to forego early switching opportunities becomes overwhelming. On the other hand, when there is an opportunity for a coordinated switch early on and there is a credible threat that foregoing a switch is likely to imply finding oneself in the incumbent when the switching process has becomes very slow, users may accept switching even when the entrant platform is socially less efficient than the incumbent. There can thus be switching equilibria where consumers coordinate on an inefficient entrant. The later could occur, for instance, when a platform credibly commits to a limited capacity.

We now present some examples of technologies that could motivate potential equilibrium switching processes and show how these illustrate the results above. All examples assume utility is linear and $r = 0$ (unless otherwise specified).

Example 3. Suppose that the rate of switching increases with the number of users who have already switched. For instance, let $1 - h(t) = m(t)$ and $\frac{dm(t)}{dt} = am(t) + b$ for $a > 0, b > 0$ and $m(0) = 0$. This yields $m(t) = \frac{b}{a}(-1 + e^{at})$ and $T = \frac{1}{a} \ln\left(\frac{Ma}{b} + 1\right)$. Defining $x \equiv \frac{b}{a}$, switching requires $k^\star = v \frac{1+(2x+1)\left(1-(x+1)\ln\left(\frac{x+1}{x}\right)\right)}{1-(x+1)\ln\left(\frac{x+1}{x}\right)}$. L'Hopital's rule then yields $\lim_{x \to \infty} k^\star = \frac{1}{3}$ (as in Example 1) and $\lim_{x \to 0} = 1$. Therefore incumbent advantage decreases in the baseline switching rate b and increases in the momentum parameter a, in accordance with the logic of Corollary 1.

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Example 4. Suppose switching opportunities arrive following a Poisson process with intensity s. Then $h(t) = e^{-st}$ and $T = \infty$. With $r = 0$, switching occurs if $\frac{k+v}{2v} \geq \frac{1}{2} \Leftrightarrow k \geq 0$. For $r \neq 0$, switching occurs if $k \geq k^\star = \frac{rv}{2s+r}$. Notice that incumbency advantage increases with the discount rate r and decreases with the arrival rate s because the latter gives the process "negative momentum" making $h(t)$ move concave: it implies a large coordinated switch early on and then a long period of very slow switching.

Example 5. Suppose that pairs of users meet with a fixed probability $\frac{a}{2}$ and that, when two users from opposite platforms meet, this gives the user in the incumbent an opportunity to switch to the entrant. If $m(t) = 1 - h(t)$, the meeting probability is $am(t)(1 - m(t)) = \frac{dm(t)}{dt}$. This differential equation, along with the boundary condition $m(0) = m_0$ solves to the Logistic equation $m(t) = \frac{1}{1+\left(\frac{1}{m_0}-1\right)e^{-at}}$, and $T = \infty$. Tipping occurs when $\frac{k-v}{2v} \geq \frac{m_0-1}{\ln\left(\frac{1}{m_0}\right)}$. The incumbent advantage is intuitively decreasing in the initial mass in the entrant, m_0. By L'Hopital's Rule we obtain $\lim_{m_0 \to 0} k^\star = v$ and $\lim_{m_0 \to 0} k^\star = -v$.

3 A Model with Heterogeneous users

3.1 A Two-Type Model

We now consider the effect of user heterogeneity in endogenously determining the switching process through user choices. In this case, incumbency advantage is no longer determined by the incentives of users deciding at $t = 0$. Some users may reject switching entirely, or accept only after a sufficient mass of users has already switched.

We consider a setup similar to that of Section 2. However, consumers have $k = k_H$ in proportion p, and $k = k_L$ in proportion $1 - p$, with $k_L < k_H$. Instantaneous utility in the entrant is $v(1 - h(t)) + k$, while in the incumbent is $vh(t)$. We assume users are given the opportunity to switch following a Poisson process with intensity s.[8]

The switching process will be endogenously determined by user choices. Types k_L choose the time $t_L^\star$ at which they begin accepting opportunities to switch. For

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[8]A Poisson process is a reasonable micro-foundation of the switching technology where users have at each instant, an independent and constant probability of being given the opportunity to switch. Moreover, a Poisson switching technology is a natural benchmark since the homogeneous users of Section 2 switch when $k \geq 0$, that is, when the entrant platform is Pareto optimal.

instance, $t_L^\star = 0$ if they accept all opportunities to switch and $t_L^\star = \infty$ if they never accept. We assume that all types k_L agree on $t_L^\star$. Given $t_L^\star$, the mass in the incumbent $h(t)$ evolves according to

$$1 - h(t) = \begin{cases} 1 - h_H(t) = p(1 - e^{-st}) & , 0 \leq t \leq t_L^\star \\ 1 - h_L(t) = p(1 - e^{-st}) + (1 - p)\left(1 - e^{-s(t - t_L^\star)}\right) & , t_L^\star \leq t \end{cases}.$$

We assume, for now, that types k_H find it profitable to accept switching at any $t \geq 0$. We can then obtain $t_L^\star$ as the instant when a user of type k_L is indifferent between switching and not, given that not switching will still imply that other types k_L will start switching at $t = t_L^\star$. We then obtain the following result.

Lemma 2. *Types k_L accept switching opportunities when the mass of users in the entrant is at least $X_L = -\frac{k_L}{v}$, and only switch if $k_L > -vp$.*

Proof. For $t_L^\star \leq t$, a user of type k_L finds herself in the incumbent platform with probability $P_L = e^{-s\left(t - t_L^\star\right)}$ and finds herself in the entrant with probability $1 - P_L$. Switching at time $t = t_L^\star$ yields utility $\int_{t_L^\star}^{\infty} [v(1 - h(t)) + k_L]\, dt$. Foregoing a switch at $t = t_L^\star$ yields $\int_{t_L^\star}^{\infty} [P_L v h_L(t) + (1 - P_L)[v(1 - h_L(t)) + k_L]]\, dt$. Indifference at $t = t_L^\star$ implies $-\frac{1}{s}\left(k_L + pv - e^{-st_L^\star}pv\right) = 0 \Leftrightarrow \frac{k_L}{pv} + 1 = e^{-st_L^\star}$. Plugging this into the expression for $h(t)$, we find that types k_L start switching when the mass in the new platform is $X_L = p\left(1 - \left(\frac{k_L}{pv} + 1\right)\right) = -\frac{k_L}{v}$. Moreover, types k_L must be at least willing to move in the limit when all types k_L have moved $(p > X_L)$, which implies $k_L > -vp$. $\square$

Intuitively, $t_L^\star$ increases with the importance of platform externalities (v) and with the proportion of types k_H because the rate at which $m(t)$ increases is proportional to initial mass in a Poisson switching process, so a larger p increases the marginal benefit of waiting. Types k_L delay the time at which they start accepting opportunities to switch if they have a preference for the incumbent $(X_L > 0 \Leftrightarrow k_L < 0)$. If k_L preferences for the incumbent are sufficiently strong $(k_L < -vp)$, these types will never switch. Finally, notice that types k_L start switching when their instantaneous utility in the entrant platform becomes weakly positive: $v\left(-\frac{k_L}{v}\right) + k_L = 0$.

We can now determine the incumbency advantage in this market: the level of k_H such that, taking X_L as given, types k_H choose to switch at $t = 0$. We obtain the following result.

Lemma 3. *If types k_L switch eventually ($k_L > -vp$), types k_H switch at $t = 0$ if the average preference in the economy favors the entrant ($0 < pk_H + k_L (1 - p)$). If types k_L never switch, types k_H switch at $t = 0$ if $v(1 - p) \le k_H$.*

Proof. Suppose $k_L > -vp$. If types k_H switch at $t = 0$, they are in the incumbent with probability $P_H = e^{-st}$. Switching at $t = 0$ yields $\int_0^\infty P_H [v(1 - h(t)) + k_H] dt$. Foregoing yields $\int_0^\infty [P_H v(h(t)) + (1 - P_H)(v(1 - h(t)) + k_H)] dt$. Indifference implies $\frac{1}{s}((+e^{-st_L^\star} - 1)(p - 1)v - k_L) = 0$. Re-arranging and using $\frac{k_L}{pv} + 1 = e^{-st_L^\star}$ yields the first result.

If $k_L < -vp$, following the same indifference argument described above, the enthusiastic types k_H will switch at $t = 0$ knowing that types k_L will not switch if $\int_0^\infty P_H [P_H vh_H(t) - v(1 - h_H(t)) - k_H] dt < 0$, which yields $v(1 - p) \le k_H$. $\quad\square$

We can now characterize the set of possible switching equilibria.

Proposition 3. *The possible equilibria of the game where consumers switch are:*

1. If $0 < k_L$ and $0 < k_H$, there exists a simultaneous switching equilibrium: both types accept opportunities starting at $t = 0$ and all users switch to the entrant.

2. If $k_L > -vp$ and $0 < pk_H + k_L (1 - p)$, there exists a staggered switching equilibrium: types k_H switch immediately while types k_L delay their switching, but both types eventually switch to the entrant.

3. If $k_L < -vp$ and $k_H > v(1 - p)$, there exists a 2 platform equilibrium: then types k_L do not switch but types k_H do, leading to two platforms in equilibrium.

4. Otherwise neither type switches.

Proof. The result follows from combining Lemmas 2 and 3. $\quad\square$

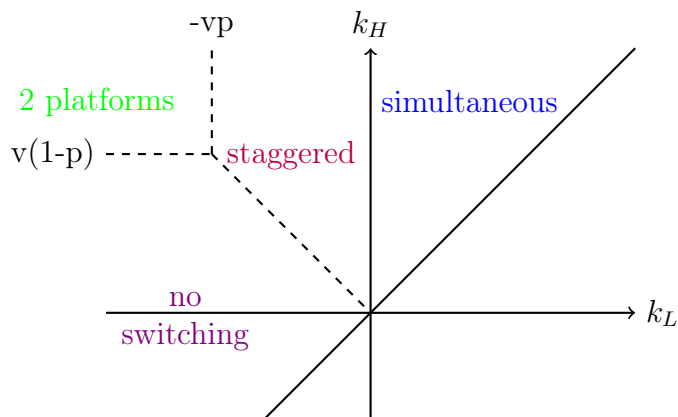The figure below illustrates graphically the four kinds of equilibria that can arise in this model.

Figure 2: 3 possible types of switching equilibria for $p = \frac{1}{2}$.

In sum, for all users to switch, two conditions are necessary. First, the entrant must be preferred to the incumbent on average. If $k_L < 0$, the most eager types have to be sufficiently eager (k_H large enough) to compensate this lack of enthusiasm and they must be in a sufficiently high proportion (p large enough). For instance, if types are balanced ($p = \frac{1}{2}$), we must have $-k_L < k_H$; if $p = 1$, we must have $k_H > 0$ as in Example 4.

However, the enthusiasm of k_H types can only go so far. In a population of fixed size, there is a maximum value of platform externalities, and therefore a maximum compensation that the k_L types can receive from joining the entrant platform. The second condition required for all users to switch is that types k_L cannot dislike the entrant platform so strongly that the entire platform value of types k_H is too little to compensate types k_L for the negative value of k_L. Thus, if preferences are sufficiently polarized, even a large average preference for the entrant will result in a 2-platform equilibrium.

3.2 The Domino Effect

As we saw above, if preferences are sufficiently polarized, it is hard to sustain an equilibrium where everyone switches to the entrant platform, even if average preferences in the economy favor the entrant. We now extend the model to include a mass of individuals with neutral preferences $k_M = 0$ who can act as a link between users of types k_H and k_L. These neutral types can then facilitate a "domino" effect

that leads all users to switch even under conditions when that would not occur in the environment of Sub-Section 3.1.

We now consider three kinds of users, with types $k_L < k_M = 0 < k_H$. We normalize the total population to 1 and assume that type k_M is in proportion $q \in [0, 1]$, while types k_H and k_L are both in proportion $\frac{1-q}{2}$. That is, q is a measure of how concentrated preferences are. Otherwise the setup is as in Sub-Section 3.1. Users are given opportunities to switch following a Poisson process with intensity s. For $i \in \{H, M, L\}$, each type k_i chooses her preferred time $t_i^\star$ at which to start accepting opportunities to switch. In equilibrium, the number of people in the incumbent are endogenously determined according to the process

$$1 - h(t) \begin{cases} 1 - h_H(t) = \frac{1-q}{2}(1 - e^{-st}) & 0 \le t \le t_M^\star \\ 1 - h_M(t) = \frac{1-q}{2}(1 - e^{-st}) + q\left(1 - e^{-s(t-\theta_M)}\right) & t_M^\star \le t \le t_L^\star \\ 1 - h_L(t) = \frac{1-q}{2}(1 - e^{-st}) + q\left(1 - e^{-s(t-\theta_M)}\right) + \frac{1-q}{2}\left(1 - e^{-s(t-\theta_L)}\right) & t_L^\star \le t \end{cases}$$
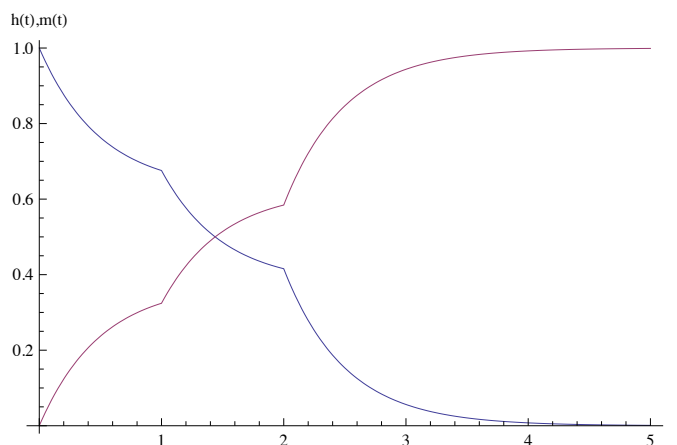

Figure 3: $h(t)$ and $m(t)$ for $\theta_M = 1$ and $\theta_L = 2$.

We adopt a same procedure as in the previous sub-section to obtain the following intermediate results.

Lemma 4. *Assuming types k_H and k_M switch eventually, types k_L start accepting opportunities to switch when the mass of users at the entrant is $X_L = -\frac{k_L}{v}$. Equivalently, types k_L switch only if $k_L > -v\frac{1+q}{2}$.*

Proof. Assuming types k_H start switching at $t = 0$ and types k_M start switching at

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$t_M^\star$, we compute $t_L^\star$. For $t \geq t_L^\star$, users k_L find themselves in the incumbent platform with probability $P_L = e^{-s(t-t_L^\star)}$. Types k_L start switching at a time $t_L^\star$ at which their are indifferent about switching and foregoing an opportunity to switch, which implies $0 = \int_{t_L^\star}^\infty P_L \{vh_L - v(1-h_L) - k_L\} dt$, or $\frac{v + \left(2e^{st_M^\star}-1\right)qv}{2k_L+v+qv} = e^{st_L^\star}$. The mass in the entrant which users of type k_L require to start to accept switching (X_L) is $X_L = \frac{1-q}{2}(1 - e^{-st_L^\star}) + q\left(1 - e^{-s\left(t_L^\star - t_M^\star\right)}\right) = -\frac{k_L}{v}$. Types k_L must be willing to move at least in the limit when all the k_H and k_M types have moved, or $X_L < q + \frac{1-q}{2} \Leftrightarrow k_L > -v\frac{1+q}{2}$. $\qquad\square$

This mirrors what was obtained in Sub-Section 3.1. If types k_M delay the time at which they begin accepting to switch, types k_L also delay switching until their instantaneous utility in the entrant platform becomes weakly positive $(vX_L + k_L = 0)$. As in Sub-Section 3.1, the lower is k_L, the more users must have switched before k_L types start to accept switching. If $k_L < 0$, as assumed, then $X_L > 0$ so types k_L do not start switching immediately.

Importantly, increasing the concentration of preferences (q) relaxes the constraint required for types k_L to be willing to switch at all $(X_L < q + \frac{1-q}{2})$. For $q = 0$, we require $k_L > -\frac{v}{2}$, but for $q = 1$ we only require $k_L > -v$. Intuitively, increasing q increases the mass of types more enthusiastic than types k_L and thus increases the value of platform externalities in the entrant for types k_L to enjoy. It is by increasing the proportion of types k_M more than it decreases that of types k_H that increasing the concentration of types facilitates the domino effect that ultimately broadens the range of parameter values for which switching is possible.[9]

Lemma 5. *Assuming that types k_L switch eventually $(k_L > -v\frac{1+q}{2})$, types k_M choose θ_M such that $X_M = \frac{1-\sqrt{1-2X_L(1-q)}}{2}$. Moreover, types k_M switch only if types k_L switch.*

Proof. Types k_M take the choice of X_L as given from above and choose θ_M such that they are indifferent about starting to switch at $t = t_M^\star$. In the period $\theta_M \leq t$, types k_M find themselves in the incumbent with probability $P_M = e^{-s(t-t_M^\star)}$. Indifference at $t = t_M^\star$ implies $0 = \int_{t_M^\star}^{t_L^\star} P_M [vh_M - v(1-h_M) - k_M] + \int_{t_L^\star}^\infty P_M [vh_L - v(1-h_L) - k_M]$. This implies $e^{2st_M^\star} = e^{st_L^\star} \Leftrightarrow t_L^\star = 2t_M^\star$. Using this condition and the expression for $e^{st_L^\star}$ found above, we obtain a system of two equations and two unknowns. Solving the

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[9]This constraint is similar to the one obtained in Sub-Section 3.1 $(k_L > -vp)$, because the proportion of types $k_i > k_L$ types was p in that case and is $\frac{1+q}{2}$ here.

system and choosing the positive solution yields $e^{2st_M^\star} = e^{st_L^\star} = \frac{qv+\sqrt{v^2-2k_L v(q-1)}}{2k_L+v+qv}$. [10] This implies that the mass required by types k_M before they accept to switch is $X_M = \frac{1-q}{2}(1-e^{-st_M^\star}) = \frac{1-\sqrt{1-2X_L(1-q)}}{2}$ where $X_L = -\frac{k_L}{v}$ as determined above.

When types k_L don't switch and types k_H switch at $t = 0$, types k_M choose $t_M^\star$ by solving $0 = \int_{t_M^\star}^\infty P_M \{vh_M - v(1-h_M) - 0\}\, dt$, which implies $e^{-st_M^\star} = 0 \Leftrightarrow t_M^\star = \infty$. Intuitively, types k_M are indifferent about switching only for $t_M^\star = \infty$ because $k_M = 0$ and they obtain the same amount of platform externalities in either platform, but utility is lost during the switching process which makes the incumbent preferable. $\square$

Notice that $X_M > 0$ because the interior of the square root is less than unity. [11] Importantly, X_M is decreasing in q. For $q = 1$, $X_M = 0$ so types k_M start switching immediately as in the previous sub-section. However, as q decreases, the dispersion of preferences increases and types k_M begin to delay the time at which they start switching, so X_M increases. It is because X_M is decreasing in q that q also relaxes the constraint on the k_L for switch switching occurs.

Finally, we can determine the level of k_H required to start switching at $t = 0$, assuming that the other types choose to start accepting switching opportunities eventually. The result mirrors that of Sub-Section 3.1 and is formalized as follows.

Lemma 6. *If $t_L^\star < \infty$ and $t_M^\star < \infty$, types k_H accept switching opportunities for all $t \geq 0$ if $k_H \geq -k_L$. If types k_M, k_L do not switch, types k_H switch if $k_H > \frac{v}{2}(q+1)$.*

Proof. If switching starts at $t = 0$, a user of type k_H finds herself in the incumbent with probability $P_H = e^{-st}$. Then types k_H are indifferent at $t = 0$ if $0 = \int_0^{t_M^\star} P_H 2vh_H dt + \int_{t_M^\star}^{t_L^\star} P_H 2vh_M dt + \int_{t_L^\star}^\infty P_H 2vh_L dt - (v + k_H)\int_0^\infty P_H dt$. This implies $\frac{1}{2}v\left(1 + q + e^{-st_L^\star}(q - 1) - 2e^{-st_M^\star}q\right) = k_H$. Using the results above for $t_M^\star$ and $t_L^\star$ yields the first result.

If $0 > \int_0^\infty P_H \{vh_H - v(1-h_H) - k_H\}\, dt$, types k_H decide to switch even when types k_M, k_L do not, which yields $k_H > \frac{v}{2}(q+1)$. $\square$

The main result of this section can therefore be formalized as follows.

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[10] If k_L types switch ($k_L > -v\frac{1+q}{2}$), we have $v^2 - 2k_L v(q-1) > 0 \Leftrightarrow -\frac{v}{2}\frac{1}{1-q} < k_L$, because $1 + q < \frac{1}{1-q} \Leftrightarrow q^2 > 0$. Moreover, $2k_L + v + qv > 0 \Leftrightarrow k_L > -\frac{v}{2}(1+q)$.
[11] $1 - 2X_L(1-q) < 1 \Leftrightarrow 0 < X_L \Leftrightarrow k_L < 0$

Proposition 4. *Full switching requires $k_H \geq -k_L$ as well as $k_L > -v\frac{1+q}{2}$. Therefore, increasing q increases the set of values of (k_H, k_L) for which all users switch to the entrant.*

Proof. The result follows from Lemma 4, with Lemmas 5 and 6 showing that an increase in q does reduce the switching incentives of other users. □

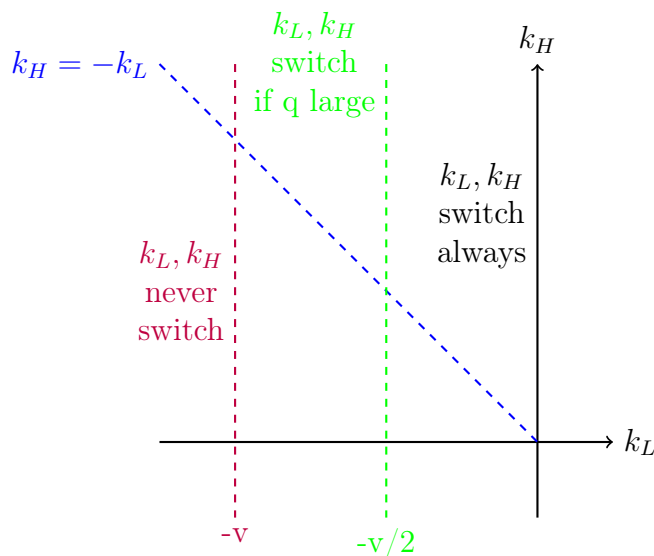The following graph and examples illustrate the domino effect.

Figure 4: The Domino Effect.

Therefore, it is still the case that the average preference in the population must be favorable to the entrant platform. Thus, full switching requires $k_H \geq -k_L$. However, it also requires $k_L > -v\frac{1+q}{2}$. Thus, the concentration of the market does not affect outcomes if the binding constraint is $k_H \geq -k_L$: the average preferences must still favor of the entrant. However, decreasing the dispersion of types does relax the constraint $k_L > -v\frac{1+q}{2}$ which can facilitate tipping if k_H is large enough. That is, conditional on the average preference being sufficiently in favor of the entrant, increasing the concentration of preferences relaxes the constraints required for the least enthusiastic types to be willing to switch and therefore makes markets easier to tip.

Example 6. Suppose $k_H = 2$, $k_L = -1$ and $v = \frac{3}{2}$. Then clearly the average preference in the economy will be in favor of the entrant platform. However, switching

will only occur if $-1 > -\frac{3}{2}\frac{1+q}{2} \Leftrightarrow \frac{1}{3} < q$. Therefore, for a concentration $q \geq \frac{1}{3}$ there is tipping, whereas for $q < \frac{1}{3}$ types k_L and k_M never switch.

4   Literature Review

Farrell and Saloner [1985] analyze an environment where a finite number of players are given a single opportunity to switch following a pre-determined order. This gives the first player the ability to disproportionately affect the decisions of others by committing to switch. In contrast, we consider a continuum of users, so an individual user's decision has no direct impact on the decision of other users, so we abstract from the "bandwagon" effects emphasized in that paper. Ochs and Park [2010] analyze an environment where a finite number of players differ in how large a platform must be before it is profitable to join. Player types are privately known so there is aggregate uncertainty about the composition of the pool of players. Players can join the platform at any period and, in equilibrium, do so sequentially using threshold strategies. In this setting, the source of frictions is the private information held by consumers, whereas in our setting consumers have no private information.

In both papers mentioned above, a user's outside option is a fixed level of utility, whereas users in our model users choose whether to switch out of an existing platform whose value decreases with time in a switching equilibrium. It is this feature of our setup that that generates the friction in our model, the free riding incentives of users to delay their switch decision, which is entirely from the papers mentioned above. As in Farrell and Saloner [1985], adoption can also be inefficient in our setting, but the source of this inefficient is not the "bandwagon" power of early movers.

Two other papers focus on issues similar to those we discuss. Sakovics and Steiner [2012] study a model without private information where a monopoly platform chooses the order in which to attract users and how much to subsidize each of them. User outside options are heterogeneous but fixed during the process of platform formation. Our model abstracts from strategic considerations by firms and focuses on user decisions. Moreover, we study circumstances where the order in which users join the platform is potentially by the distribution of user heterogeneity, rather than chosen directly by a profit maximizing platform. Cabral [2011] studies a model of competition between platforms that adjust their prices dynamically. In every period, a share of users is can re-optimize their platform choice given the current prices and the mass

of users in each platform. In contrast to this paper, we abstract from firm pricing decisions. On the other hand, our focus on the technology of the switching process is absent from that paper.

5 Conclusion

This paper studies the dynamic formation of competing platforms and the factors determining incumbency advantage in platforms markets. We link incumbency advantage to the properties of the process according to which users switch platforms in equilibrium, showing that the degree and timing of switching opportunities is paramount in determining incumbency advantage. We then examine the role of user heterogeneity and find that switching occurs if the average user in the economy favors the entrant platform and if preferences are sufficiently concentrated.

Our analysis abstract from strategic decisions by platforms. While k can be interpreted as a difference in prices or qualities between platforms, it is left as exogenously determined and fixed over time. Letting the firms determine k (for instance, prices) would allow us to characterize how the switching technology interacts with firm strategies. One could then determine to what extent an entrant might be willing to loose revenue by lowering its price in order to incentivize switching.[12] Along the same lines, it would be interesting to determine the circumstances under which a entrant might commit to a restricted capacity in order to increase the cost of foregoing early switching opportunities and thereby reducing incumbency advantage.

Moreover, we have assumed a particular form of heterogeneity which implies a specific order in which users would join the platforms. In reality, users are likely to differ in more than one dimension. For instance, allowing for heterogeneity in both v and k would enrich the model substantially. In this case, we conjecture that users could still be ordered in the way they switch platforms in the following way: the first to switch would be users who prefer the entrant and have little value for externalities, followed by users who are indifferent between the platforms and have large values for externalities, followed finally those who prefer the incumbent and do not have a large value for platform externalities.

Our paper also abstracted from the effect of switching processes on welfare. In particular, it would be interesting to compute the welfare lost during the transi-

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[12]On this topic, see Weyl [2010] and Cabral [2011].

tion between platforms for different switching processes and determine, for a given switching process, how large does k have to be for switching to be socially optimal. Similarly, it would be interesting to determine the circumstances under which the 2 platform equilibria described in Section 3 are socially optimal despite leading to a lower amount of platform externalities.

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